New Method to Measure Metallurgical Length and Application to Improve Computational Models

In this work, a "sensor-less" method infers metallurgical length by observing when a particular roll first starts turning during small-step increases in casting speed. This method was applied to the thin-slab caster at Nucor Steel–Decatur LLC, and the trial results were used to calibrate a real-time computational heat transfer model.

Authors

Bryan Petrus
process automation engineer, Nucor Steel–Decatur LLC, Trinity, Ala., USA
bryan.petrus@nucor.com

Danny Hammon
caster mechanical technician, Nucor Steel–Decatur LLC, Trinity, Ala., USA
danny.hammon@nucor.com

Megan Miller
material handling supervisor, Nucor Steel–Decatur LLC, Trinity, Ala., USA
megan.miller@nucor.com

Bob Williams
material handling supervisor, Nucor Steel Tuscaloosa Inc., Tuscaloosa, Ala., USA
bob.williams@nucor.com

Adam Zewe
meltshop manager, Nucor Steel Decatur–LLC, Trinity, Ala., USA
adam.zewe@nucor.com

Zhelin Chen
graduate research assistant, University of Illinois at Urbana-Champaign, Urbana, Ill., USA
zchen61@illinois.edu

Joseph Bentsman
professor of mechanical engineering, University of Illinois at Urbana-Champaign, Urbana, Ill., USA
jbentsma@illinois.edu

Brian G. Thomas
C.J. Gauthier Professor of Mechanical Engineering, University of Illinois at Urbana-Champaign, Urbana, Ill., USA
bgthomas@illinois.edu

As of 2013, more than 90% of steel cast in the world was made via continuous casting.[1] This process is characterized by a continuous flow of liquid steel into the caster that solidifies as it moves through the machine and exits the caster fully solid. Knowing where the steel in the caster (the strand) becomes fully solid is critical to safety, productivity and quality goals. The steel industry and academic researchers have spent a great deal of effort to develop methods, involving either measurements or predictive models, for this purpose. The task is difficult in part because the very definition of "fully solid" changes depending on the particular goal, and this affects which measurement or modeling technique is best suited.

Introduction

Metallurgical length (ML) is generally defined as the distance from the liquid level in the continuous casting mold to the location in the caster where the cross-section finally becomes fully solid. The rest of this introduction defines the terminology used in this paper. Then, a brief survey of previous methods to measure ML is given. A new approach, published here for the first time, is described in detail, including its use at Nucor Steel–Decatur LLC. This method requires no additional equipment or instrumentation. Next, a brief discussion of mathematical models to predict ML is provided. A particular dynamic computational model, Consensor, has been calibrated based on the measurements from the results of the new approach applied at the Nucor Steel–Decatur LLC caster. In addition, the Consensor model is validated against measurements of ML from the literature on a conventional thick-slab caster.

Definition of Metallurgical Length — The difficulty in defining ML is that, as an alloy, steel does not have a single freezing temperature from entirely liquid to entirely solid. Instead, steel solidifies over a range of temperatures, when it is in what is often called a "mushy zone" and consists of a complex dendritic microstructure containing both solid and liquid. This is illustrated in Figure 1. Two possible definitions of "solid" steel are based on the temperature when it first starts solidifying, called the liquidus temperature, or the temperature when no liquid remains, called the solidus

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temperature. Many process goals are determined by conditions in the mushy region, which is neither completely solid nor completely liquid as the steel dendrites grow out from the completely solid shell into the liquid in the middle of the strand.

For example, a serious concern is containment of the liquid steel. At the exit of the mold, the steel shell must be sufficiently thick to prevent the liquid from breaking out, which would be catastrophic. Even if it does not break out, the liquid steel pool is under pressure from gravity, causing it to push out against the steel shell. This "bulging" caused by the ferrostatic pressure is actually severe enough to deform the shell catastrophically in a slab caster, if it is not sufficiently supported. Inside the caster, the steel shell is supported by the mold and later by the containment rolls (also called support rolls). However, if there is still liquid remaining when the steel exits the last containment roll, the pressure will cause excessive outward bulging of the shell, creating a "whale." Casting must halt until the whale is solid enough to cut apart and pull out by crane. The caster itself can be damaged, and in severe cases molten liquid steel can even escape the shell, endangering the caster operators and damaging equipment. Typically, a fraction of 70% solid is considered enough to resist whale formation.

On the quality side, the concentration of alloying elements in the liquid steel increases as the temperature decreases from solidus to liquidus according to the phenomenon of "segregation." As the strand solidifies, highly alloyed liquid is pushed toward the center of the strand. At a certain fraction of solid, the dendrites interlock enough to prevent the liquid from flowing, trapping pockets of high-alloy material and creating defects called "centerline segregation." Some casters conduct "soft reduction" practices, which aim to prevent this by shrinking the roll gap over the region where the centerline is between the liquidus and the solidus to counteract the shrinkage forces, which tend to draw out the high-alloy liquid and concentrate it in centerline pockets.

Measurements

Because of its importance, methods to measure ML have been studied since the earliest days of commercial continuous casting. In this section, previous measurements related to ML are classified into two categories. The first type is a general measurement
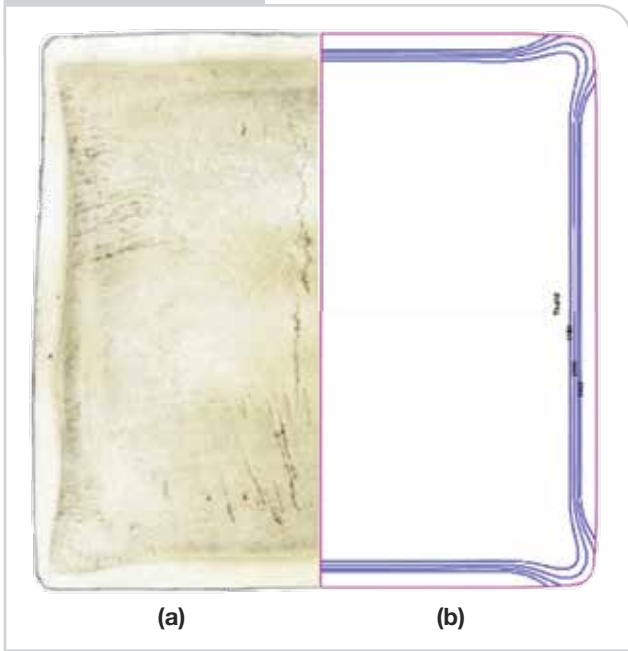

Figure 1
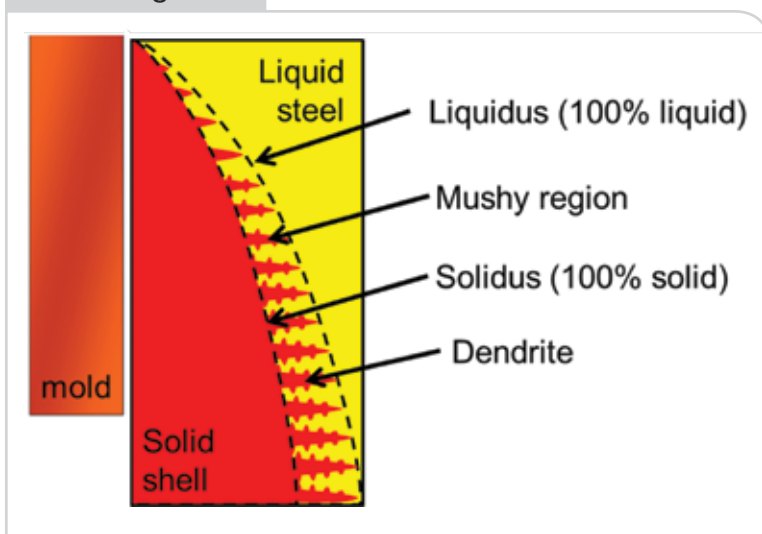
Solidification of steel in a continuous caster.

of shell thickness. These methods are not necessarily direct measurements of ML, since they may be applied anywhere along the caster. Even if they are not used near final solidification, they can be used to eventually predict ML through the use of models, as discussed in the next section. The second type of method directly measures ML by detecting the location of final solidification inside the strand.

Measurements of Shell Thickness — One of the first methods to measure shell thickness was accomplished by Brimacombe and co-workers[2] in 1974. Radioactive gold was injected into the mold at the meniscus, where it could dissolve and spread throughout the liquid, but not the solid. The steel was then cut into sections, and "autoradiographs" were taken from the radiation emitted. This gave one of the earliest measurements of shell thickness, but with several important drawbacks. From a technical standpoint, the radioactive liquid may not spread completely throughout the liquid cavity, or it may not penetrate the mushy zone, and so the autoradiograph shows a larger shell thickness than is realistic. From a practical standpoint, this method produces a slab of radioactive steel that is dangerous to handle, impossible to sell and costly to clean up.

Sulfur printing is a related method that shares the general idea of dropping a tracer into the liquid steel, but uses chemical etching rather than radiation to provide the image. As the name implies, the tracer in this case is sulfur, which shows up brownish-yellow after etching. Since the sulfur can diffuse only where there is liquid, the solid and liquid areas of the steel can be clearly seen on the sulfur print. An example using this method is given in Reference 3. Figure 2 is an image from that paper, comparing a sulfur print
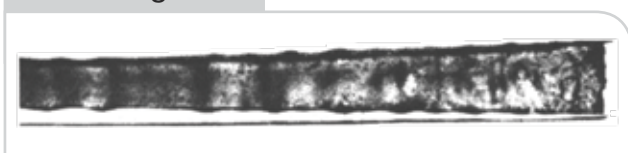
Figure 2

Sulfur printing showing etched cross-section of a billet (a) and temperature contours from finite element method (FEM) simulation (b).[3]

Figure 3
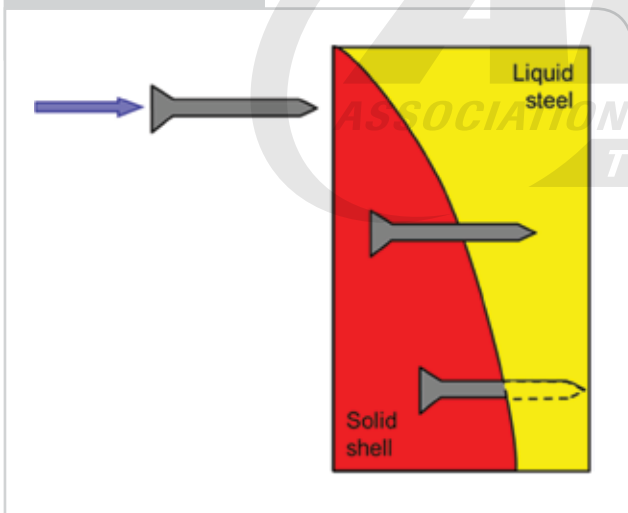
Nail-shooting method.

Figure 4

Steel shell remaining after a breakout.[5]

to the results of a two-dimensional transient finite element method (FEM) model. Note that the shell thickness seen in the sulfur etch appears to match well with the predicted location of the solidus temperature calculated by the model.

Another method that indicates the shell thickness on cross-sectioned samples relies on altering the strand mechanically, rather than chemically. This was described in Reference 4. Small shims were dropped onto the broadface of a conventional slab caster in between two support rolls. As the strand traveled, the shim was squeezed between the steel shell and the support roll, deforming the shell enough to cause internal cracks. Cracks form where the steel still contains liquid, near the solidification front. These cracks fill with segregated liquid and can be seen and measured in a cross-section of the strand.[4]

Another mechanical method to measure shell thickness is nail shooting. An illustration is shown in Figure 3. Using a nail gun, a metal nail is shot into the strand with enough force that the nail extends partially into the liquid steel. The hot liquid melts the nail, leaving intact only the portion embedded in the solid shell. The volume of the strand holding the nail can be cropped out, cross-sectioned, examined to determine where the nail was melted and measured to determine the thickness of the solid shell where the nail was shot.

There is one more type of "trial" that is important to mention: catastrophic failures. For example, when liquid steel escapes from the solid shell in caster, the event is called a "breakout." These events are costly, dangerous and should be avoided as much as possible. However, once one has occurred, the most sensible thing to do is to learn as much as possible from the evidence remaining. This includes the emptied steel shell, an example of which is shown in Figure 4. The shell, once cut into sections, can easily be measured. As shown in the figure, the inside surface of a breakout shell is usually smooth. During the breakout, however, the exposed mushy zone should consist of solid dendrites sticking up into the liquid. The smooth surface of the breakout shell implies that liquid is trapped between the dendrites and will eventually solidify. Thus, the surface of a breakout shell should roughly correspond to the liquidus temperature in the strand.

The methods discussed above are designed to measure shell thickness. As such, they often give more than a simple measure of ML, namely a profile of the shell thickness. Specifically, autoradiographs provide the profile in the longitudinal plane. Sulfur prints typically provide the profile in the transverse plane. A breakout shell, depending on how it is cut up, can give either direction, although the area near the actual breakout location on the slab is usually

Figure 5
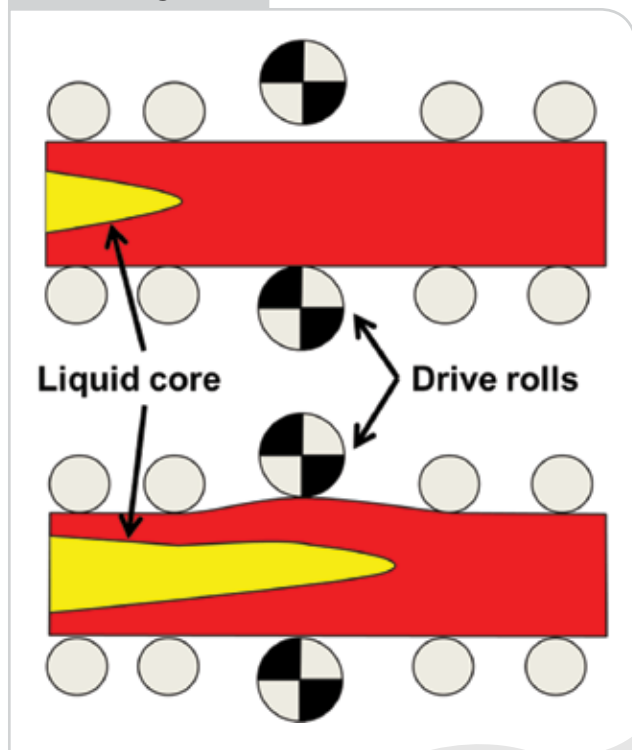
Dynamic metallurgical length (ML) measurement at Nucor Steel–Decatur LLC.

not representative of regular casting conditions. Also, these methods are all destructive methods that require cutting apart the finished slab, and therefore cannot determine ML during actual casting. However, these methods can be used to calibrate or validate models of shell thickness that can then be used to predict ML on-line.

Measurements of Metallurgical Length — Several other methods have been developed to measure ML directly in an operating caster. One example of this was shown in a paper describing a redesign of a caster at ArcelorMittal Burns Harbor.[6] When constructing new segments, strain gauges were added to the support rolls at various locations near the end of the caster. If liquid is contained within the strand beneath a support roll, the ferrostatic pressure pushes out against the roll, and the strain gauge reads a higher value than when the strand is solid and there is no liquid beneath the roll. By looking for sharp changes in the strain measurement, the movement of the liquid core can be monitored and located to the span between two of the instrumented rolls. Of course, this requires installation and maintenance of the strain gauges.

Toshi Hirose, an engineer at Nucor Steel–Hertford County in North Carolina, developed a method based on the same principle as the above method, but did not require any sensors other than measuring the casting speed and observing the caster itself. The caster has dynamic soft reduction capability, i.e., the gap between some of the rolls can be changed to some extent during operation. Optimizing where to taper the roll gap requires knowing the actual position of final solidification. Hirose devised a trial to locate the liquid core. First, a segment known to be beyond the metallurgical length was opened up slightly. As with the strain gauges, when no liquid is beneath a roll, there is no force pushing the shell against the rolls. Contact between the shell and rolls was lost, so the rolls in the chosen segment stopped turning. Then, the casting speed was slowly increased. Eventually, the liquid core moved underneath the first roll of the segment and the pressure pushed the shell against the roll, causing the roll to start turning again. This indicated that the metallurgical length was approximately at the location of that roll at that casting speed.

The casters at Nucor Steel–Decatur LLC are not capable of dynamic soft reduction. However, this trial was adapted to run at the Decatur facility using the new method illustrated in Figure 5. Although the segments themselves cannot be opened up during casting, the drive rolls can be. This was designed to allow a gap measurement device to pass through the caster without the hydraulic force on the drive roll crushing the sensitive electronics of the device. For the new trial, the drive roll was moved far enough away from the caster that the roll lost contact with the steel surface. The motor was then de-powered so the roll eventually came to a stop. As in the Hertford County trial, the casting speed was increased in small steps until the roll started turning again. This indicated that there was liquid beneath the roll, bulging the shell back into contact with the drive roll.

Figure 6 shows the data collected during the trial. The blue line is the casting speed. The purple line is the predicted ML from Consensor, a real-time computational model that will be discussed in more detail in the next section. The figure shows how, as the speed increases, the predicted metallurgical length increases after a delay. Starting from the beginning of the trial, referring to the numbers in Figure 6:

1. The casting speed was lowered to 115 inches/minute to ensure that the metallurgical length was before the roll to be lifted. A steady-state starting condition was achieved by lifting the roll and waiting for it to stop moving and for the dynamic prediction of ML by Consensor to reach a constant value.
2. The casting speed was increased in steps of 5 inches/minute. The speed was held constant for 5 minutes after each increase, which

Figure 6
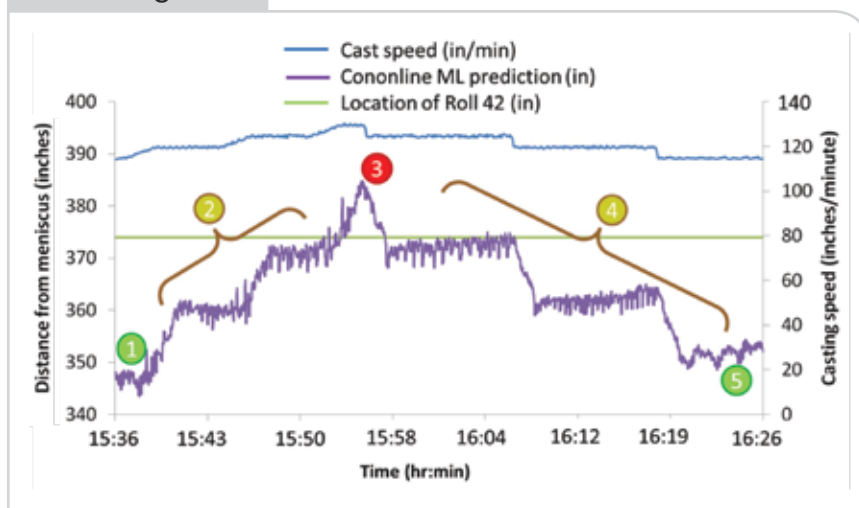
Casting conditions and predicted ML during Nucor Steel–Decatur LLC trial.

was more than enough time to allow the caster to reach steady state. During this time, the roll sometimes shifted, but never continuously. This may have been caused by local dynamic bulging, asymmetrical thermal distortion of the roll, intermittent scale or some other cause.

3. At a casting speed of 130 inches/minute, the roll began turning continuously. To prevent this bulging from causing centerline segregation and cracking issues in the steel, the speed was immediately lowered.

4. In order to check for hysteresis, the speed was returned to 115 inches/minute, again in 5 inches/minute increments. No difference was seen between the roll behavior after increasing and after decreasing casting speed.

5. The trial concluded and normal production resumed.

The conclusion of this trial is that the metallurgical length, under the casting conditions of the trial, reaches the location of the roll at a speed between 125 and 130 inches/minute. This information was then used to calibrate the computational model Consensor, as discussed in the next section.

Model Prediction of Metallugical Length

The difficulty and expense of measuring ML have led to many attempts to predict ML using models. The simplest of these are "K-factor" models, used commonly throughout the industry. Similarity analysis of a simple mathematical description of solidification[7] predicts that the thickness, s, of the solid in a solidifying material theoretically grows as:

$$s = K\sqrt{t}$$

(Eq. 1)

where

K = factor giving the model its name and
t = time from when the metal started solidifying.

For a continuous casting with strand thickness L and casting speed V, this simple model gives a prediction of the ML:

$$ML = \frac{L^2}{4K^2}V$$

(Eq. 2)

This is based on calculating the time for the shell thickness to grow to $L/2$, since the caster is solidifying from both sides, and the distance ML that the steel moves from the top of the caster in that time.

One obvious issue with this model is that the only two casting conditions included are casting speed and strand thickness. While these are the two most important conditions determining ML, other important conditions, such as mold water cooling, secondary spray cooling, pour temperature and steel grade, are all hidden inside the factor K itself. In practice, different K-factors can be determined for different conditions. Another less obvious problem is that some important, and unlikely, assumptions are hidden in the underlying formulation of the model. First, as discussed earlier, steel has liquid, solid and mushy zones, not a single shell thickness. This is typically accounted for by using different K-factors for various solidification temperatures. Second, the model that produces this relationship assumes a constant temperature at the surface of the strand, which is generally not true. It also assumes a constant temperature in the liquid pool, which is a reasonable approximation. Third, the model assumes an infinite amount of liquid steel. In reality, as final solidification approaches, the shell

Figure 7
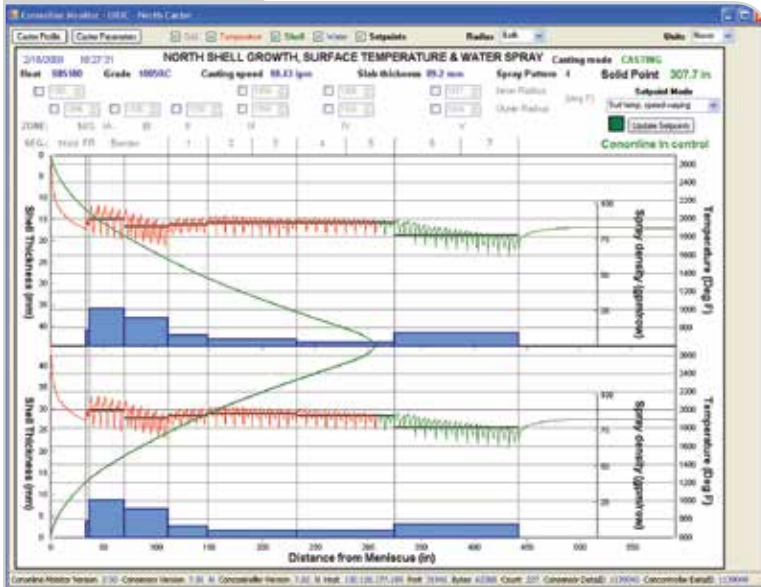
Example temperature and shell thickness profile predicted by Consensor during casting at Nucor Steel–Decatur LLC.

grows faster than the square root of time as heat is removed in both directions. All of this means that a value of *K* calibrated to match a breakout shell, for example, will fail to correctly predict ML, and vice versa. However, for much of the history of continuous casting, the *K*-factor model had an essential advantage over other models: its simplicity means that it can be calculated during production in response to changing casting conditions.

Another approach is to numerically solve the heat conduction governing equation with solidification and appropriate boundary conditions. A great deal of work and research has been performed using such models, most of which are beyond the scope of this paper. These models range from simple one-dimensional heat transfer models to much more complicated three-dimensional models of heat transfer coupled with equations for mechanics, chemistry, fluid flow and other phenomena. Early in the history of continuous casting, it was simply impossible to calculate these models fast enough for use on-line in production. As computer speeds have increased and computational methods have grown more sophisticated, real-time computational models of solidification have become a reality. The earliest such model published for continuous casting was by Louhekilpi and co-workers,[8] followed by Hardin et al.,[9] and then Zheng et al.[10] The great advantage of these more fundamental models over the *K*-factor model is that they predict more than just ML. In order to compute final solidification, they must also predict shell thickness and temperature throughout the strand. Therefore, any of the measuring techniques mentioned earlier can be used to improve or validate the accuracy of a computational model. Ultimately, though, the best test of a model's ML prediction is a comparison to a direct measurement of ML.

Consensor is currently in use at Nucor Steel–Decatur LLC. The purpose of the trial discussed in the previous section was to gather data to calibrate Consensor. As such, it is worth discussing
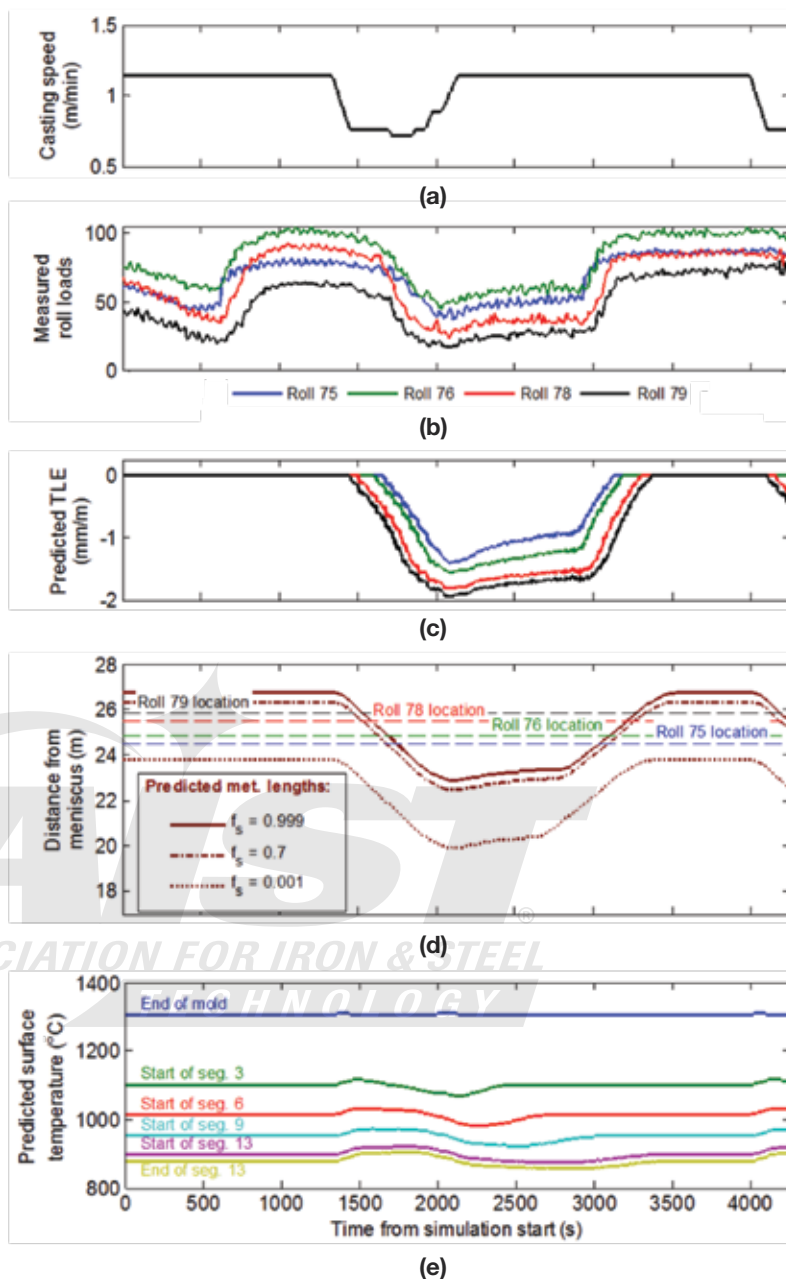
Figure 8
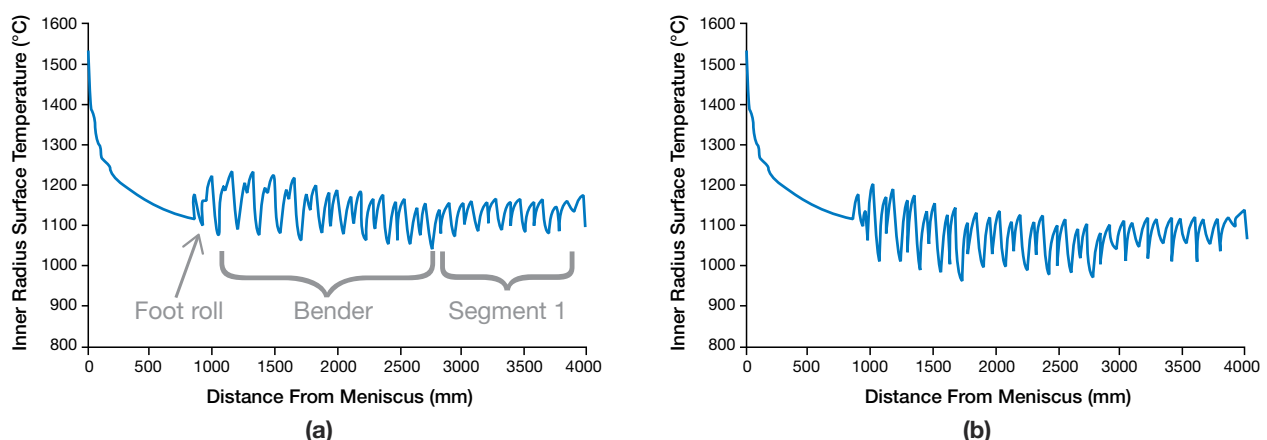
Example surface temperature profiles predicted by Consensor, before (a) and after (b) calibrating support roll conduction as a result of the ML trial.

the model in slightly more detail. Consensor is a real-time model of temperature and solidification for continuous steel slab casters, designed as part of a system to control secondary cooling sprays in the caster. A full description of this model has been published elsewhere,[11] so only a brief overview will be given here. Consensor outputs a real-time prediction of the temperature and shell thickness of the strand, using measurements of current casting conditions, including casting speed, tundish temperature, and water flowrates in the mold and secondary cooling sprays. Consensor takes advantage of the fact that, at typical continuous casting speeds, advection heat transfer (the movement of steel through the caster) dominates conduction heat transfer in the casting (axial or longitudinal) direction. This means that the computation grid does not need to extend in that direction. Hence, rather than compute a model of the entire caster, Consensor models thermally independent transverse slices of material moving through the caster at the casting speed. This slice-tracking approach produces a prediction of the temperature and shell thickness of the strand faster than a single two-dimensional model. An example prediction of Consensor is shown in Figure 7.

Figure 9

Consensor model predictions of dynamic temperature, solidification and thermal shrinkage during a series of speed changes in a caster at ArcelorMittal Burns Harbor, compared to roll loads measured by strain gauges from Reference 6.

Calibration and Validation of Computational Model Using Trial Data

Calibration of Consensor Using Nucor Steel–Decatur LLC Trial —

Gathering data to calibrate Consensor has been challenging. In fact, the very problem Consensor was meant to address is the lack of reliable temperature and solidification measurements available in continuous casters. Pyrometers inside the caster at Nucor Steel–Decatur LLC have not been reliable in the past, due to the large amount of spray water and steam in the caster making it difficult to get a consistently clear view of the strand surface. Some calibration was performed on breakout shells, but a measurement of the final solidification point is much more useful, as discussed earlier. The purpose of the trial at Decatur was to obtain such a measurement.

The trial was very useful, because Consensor initially was found to overpredict ML. Consensor originally predicted that the ML was beyond the location of

the raised drive roll at a casting speed of 125 inches/minute, instead of 130 inches/minute as actually seen during the trial. Subsequent investigation found that Consensor was greatly underpredicting the effect of cooling by the support rolls early in the caster. The original design of the Nucor Steel–Decatur LLC segments had no internal cooling in support rolls in the first few segments. Internal water cooling was later added to these rolls, but Consensor was created based on the original design and assumed little heat was removed by these rolls. Figure 8a shows the surface temperature prediction of the initial version of Consensor. Note the repeating pattern of a large drop followed by a small drop. The large drop is a water spray, and the small drop is in contact with a support roll.

Figure 8b shows the change to Consensor made as a result of this trial. The roll cooling effect in the mold foot roll, bender and first segment was adjusted to be consistent with the other rolls in the caster. The effect on surface temperature is quite small, but this had a more noticeable effect on the ML predictions, which were reduced by approximately 0.5 m.

Figure 6 shows the Consensor prediction using the recalibrated parameters as a purple line. The location of the lifted drive roll is shown as a green line. After calibration, Consensor predicts that the ML moves past the drive roll when the speed changes from 125 to 130 inches/minute, agreeing with the results of the trial.

Validation of Consensor With Strain Gauge Measurements From Conventional Slab Caster —

In the previously mentioned publication,[6] some example measurements are provided from strain gauges installed in a conventional slab (260 mm thick) caster during a series of changes in casting speed. Figures 9a and 9b show casting speed and strain gauge measurements collected from Figure 16 in the referenced paper. The casting speed initially drops, which should lead to a smaller metallurgical length. After the speed drops, there is a delay and then the strain gauge measurements step down in magnitude. This indicates the liquid core is no longer beneath the support roll. When the speed rises again, there is a longer delay before the strain gauge measurements increase back to their original values. As a measurement of ML, the results are a clear success. Based on which strain gauges are reading high and which are reading low, the ML can be localized to between two strain gauges.

This data is also very useful for validating the transient response of the Consensor model. Consensor achieves fast computation speed by assuming all heat transfer in the casting direction occurs due to advection, neglecting conduction. The Nucor Steel–Decatur LLC caster is 90 mm thick, and typical casting speeds are above 3 m/minute. In contrast, the ArcelorMittal Burns Harbor caster is 260 mm thick, and the casting speeds in the example shown are all less than 1.5 m/minute. These differences in conditions significantly test the accuracy of the model. A previously published computational model[8] with full two-dimensional grids predicted much longer transient times than Consensor predicts for conventional slab casters, suggesting that axial conduction may be significant.

Figures 9c–9e show predictions of Consensor for the speed changes in Figure 9a. To perform this simulation, the casting speed history was taken directly from the previously referenced graph in the paper and roll gaps from other figures. A simple, low-carbon steel (liquidus 1,532°C, solidus 1,515°C) was used, with a 1,550°C pour temperature. The remaining conditions were chosen to match reported steady-state metallurgical lengths given in the paper. Mold water heat removal was assumed to be proportional to the square root of the casting speed. This is the theoretical relation for constant surface temperature[7] and is close to empirical correlations previously reported.[12,13] Secondary cooling water flowrates were assumed to be linearly related to casting speed. Coefficients of proportionality were chosen to match the reported metallurgical lengths.

Thus, Consensor was calibrated to this thicker-slab caster using only steady-state measurements. There are no adjustable parameters for transient conditions. Despite that, and the simplicity of the assumed casting conditions, the model produces a remarkable quantitative match in the time domain of the relative changes in the dynamic strain gauge measurements. Specifically, the predicted timing of changes in thermal linear expansion (TLE) in the third graph of Figure 9 match very well with the timing of the changes in roll loads in the second graph. The model's somewhat ad-hoc method of predicting TLE at a particular point in the caster is discussed in detail in Reference 14. As a summary, the model predicts that when liquid can still flow freely throughout a section of the steel, there is no overall thermal contraction because liquid steel will flow into the section to fill whatever gaps are formed from local contraction, while ferrostatic pressure keeps the shell in contact with the support rolls. The strand thickness will therefore equal the roll gap. After the steel is coherent enough to prevent liquid flow (assumed to be 70%), the steel is assumed to shrink naturally, based on reported measurements in the literature.[15–17] In Figure 9, the times with 0 TLE indicate the model predicts that the steel beneath that particular support roll is not yet coherent. Once it reaches coherency, the figure shows the average TLE of the steel beneath the roll. Since the actual strain on the rolls will depend on the difference between roll gap and thermal shrinkage, and the stiffness of the

segment, this provides only a qualitative comparison to the strain measurements.

The model-predicted TLE shows a longer delay and a faster change during the speed-up than during the slow-down, which agrees with the measurements. Following the slow-down in casting speed, the measured strain and model-predicted TLE both decrease. Then, as the casting speed resumes increasing, both the strain gauge measurements and model TLE predictions dip further before rebounding back up to a steady value. By this odd behavior, the model is illustrating that the strand never completed its transient response to the steady conditions at the lower casting speed before the speed was increased again. Finally, it is important to note that if axial conduction were significant, the model predictions would not match the measurements. This validates the slice-tracking approach of the Consensor model. Further results from the calibrated and validated Consensor model will be provided in future publications.

Conclusions

It is difficult to measure metallurgical length with the amount of accuracy desired for optimal continuous caster operation. Modern methods give, at best, an accuracy of one support roll pitch and require extensive instrumentation of the caster segments. However, modern casters are significantly complicated, making it difficult for a model to reliably predict ML. Ultimately, it is recommended that the ideal approach is to combine the best aspects of both approaches. A model that is well-calibrated to a particular caster can be simpler, and hence faster and less likely to suffer from implementation errors, without sacrificing predictive power. Moreover, model calibration can synthesize the information from many different measurement methods in a fundamental and physically realistic way. At the same time, an uncalibrated and unvalidated model is less useful than an educated guess. Neither approach alone, measurement or modeling, is sufficient; they are far more useful when applied together.

Acknowledgments

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