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Online Recalibration of the State Estimators for a System with Moving Boundaries Using Sparse Discrete-in-Time Temperature Measurements

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***Abstract*—In this paper, the problem of estimation is considered for a class of processes involving solidifying materials. These processes have natural nonlinear infinite-dimensional representations, and measurements are only available at particular points in the caster, each corresponding to a single discrete-in-time boundary measurement in the Stefan problem partial differential equation (PDE) mathematical model. The results for two previous estimators are summarized. The first estimator is based on the Stefan problem, using continuous instead of discrete-in-time boundary measurements. The second estimator employs a process model that is more detailed than the Stefan Problem, but with no output injection to reduce estimation error, other than model calibration. Both of these estimation frameworks are extended in the current paper to a more realistic sensing setting. First, an estimator is considered that uses the Stefan Problem under some simplifying but practically justified assumptions on the unknowns in the process. The maximum principle for parabolic PDEs is employed to prove that online calibration using a single discrete-in-time temperature measurement can provide removal of the estimation error arising due to mismatch of a single unknown parameter in the model. Although unproven, this result is then shown in simulation to apply to the more detailed process model.**

I. Introduction

Processes involving solidification are wide-spread in manufacturing, but pose several significant challenges to traditional control theoretic methods, as they are fundamentally infinite-dimensional and nonlinear in nature. The simplest, but still accurate, model of such processes, commonly called the Stefan Problem, splits the spatial domain into separate sub-domains for the liquid and solid parts of the material. Within the sub-domains, temperature follows the usual parabolic heat-diffusion partial differential equation (PDE). The boundary between the domains moves according to conservation of energy, written as the Stefan condition in terms of the temperature gradients on both sides of the boundary.

Moreover, specific solidification manufacturing methods pose problems that are generally not considered within the field of distributed parameter control systems. Consider the process of continuous casting, which as of 2013 was used to make more than 90% of the steel in the world [1]. An illustration of this process is shown in Figure 1. Continuous casting, as opposed to traditional casting methods, keeps a constant flow of liquid metal into the machine. The metal in the caster, called the strand, cools and solidifies as it moves through the machine. Heat is removed by water cooling sprays and direct contact with support rolls. At the exit of the caster, the fully solid metal is cut into separate pieces either to be processed further or shipped directly to a customer.

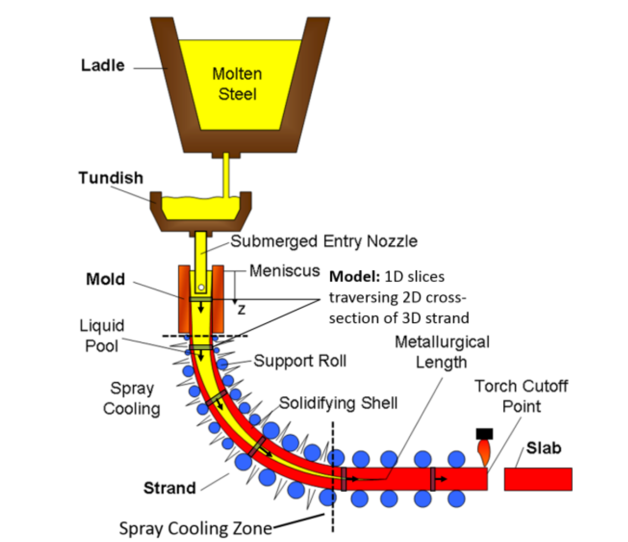

Figure 1. Illustration of continuous steel slab caster.

The basic estimation problem for this system, estimating the distributed temperature profile within the strand using only boundary measurements, is difficult enough given the nonlinear governing PDE. However, the actual measurements available are typically sparse. The support rolls and the machinery of the caster itself block access to the strand surface in much of the caster. The rest of the surface is usually being sprayed with water to cool the metal. The sprays themselves and the steam where they hit the strand interfere greatly with optical pyrometers. A typical caster will have likely one or two in the entire machine.

In Section II, a basic mathematical model of solidification is described, based on a continuous steel slab caster. The specific difficulties of the estimation problem are described. In Section III, a brief description is given of some previous work on the subject. In Section IV, a new result is described, building on the previous work to move towards an implementable estimator that uses the measurements available with enough accuracy to be used as feedback for control.

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II. Mathematical Models

Before introducing the actual partial differential equations, a short discussion is needed on scaling analysis. The discussion to follow is based on [2], where more detailed information may be found. Inside the strand of a caster, heat is transferred by two methods: diffusion and advection. The latter is heat transported by the actual movement of metal through the caster. At typical casting speeds, advection heat transfer is much faster than diffusion heat transfer, to the point where the latter is negligible in the casting direction. Rather than model the entire three-dimensional (3D) domain, reasonable accuracy is achievable by modelling a two-dimensional (2D) slice of the material as it moves down through the caster. Furthermore, in slab casters, i.e. when the aspect ratio of the 2D slice is very large, heat transfer is dominant in the smaller transverse dimension. Therefore, a one-dimensional (1D) slice gives good accuracy, and will be used for this work. For simplicity, the work in this paper will also assume the temperature is symmetric across the center of the strand. This will simplify the notation, and the results can mostly be generalized straightforwardly.

This 3D-to-1D dimension reduction is important to the present work because changing the frame of reference changes the nature of the measurement. In the full 3D reference frame, a pyrometer is a point measurement in *space*. In the 1D reference frame, a pyrometer is instead discrete in *time*, taken when the slice passes beneath the location where the pyrometer is installed. The common control theoretic concept of estimation assumes measurement over a non-zero length of time, and so many existing results cannot be directly applied to this problem.

A. The Stefan Problem

The Stefan Problem [3] models a solidifying material, in this case the moving 1D slice of the caster, by dividing it into two separate sub-domains, solid and liquid. Within each subdomain, temperature evolves according to the usual linear parabolic heat diffusion equation. The boundary between the two domains moves according to conservation of energy between the heat fluxes—proportional to the temperature gradients—on either side of the boundary and the latent heat of solidification.

Denote x to be the spatial variable, t to be the time variable, $T(x,t)$ to be the temperature and $s(t)$ - the location of the liquid-solid interface. In the equations to follow, subscripts of x and t indicate partial derivatives. In general, arguments will not be included to simplify notation. Then, the Stefan Problem is written as

$$T_t = aT_{xx}, x \in (0,s) \cup (s,L), \tag{1}$$

$$T(s,t) = T_f, s_t = -bT_x|_{x=s^-}^{x=s^+}, \tag{2}$$

where the material is solid for $x \in (0,s)$ and liquid for $x \in (s,L)$, L is the half-thickness of the slab, T_f is the melting temperature, a is the thermal diffusivity, and b is a constant related to the thermal conductivity and latent heat of solidification. For the specific 1D slice problem, the initial conditions (ICs) and boundary conditions (BCs) are

$$T(x,0) = T_0, s(0) = s_0, \tag{3}$$

$$T_x(L,t) = 0, \tag{4}$$

$$T_x(0,t) = q(t). \tag{5}$$

The boundary heat flux q will be discussed below.

One assumption will be made to simplify the problem:

(A1) $T_0(x) \le T_f, x \in (0,s_0)$ and $T_0(x) = T_f, x \in (s_0,L)$.

That is, the material is initially below the melting temperature in the solid and equal to the melting temperature in the liquid. The first condition is physically necessary. The second condition is simplistic, but not overly so. The temperature superheat (temperature above the melting temperature) in the liquid in a caster is around 25 to 50 °C, while the temperature at the strand surface is hundreds of degrees below the melting temperature. Therefore, neglecting the temperature gradients in the liquid is a common simplification in modelling continuous casters. Since this limits the temperature transients to the solid area, this simplification is sometimes called the "single-phase" Stefan Problem.

One useful consequence of this assumption is that the Stefan condition (2) simplifies to

$$s_t = bT_x(s^-,t) \tag{6}$$

The second useful consequence follows from the maximum principle for parabolic PDEs, the principle that a parabolic PDE solution attains its maximum value on the boundaries of its spatiotemporal domain. In this case, (2) and (A1) together imply that

$$T < T_f, x \in (0,s), \text{and } T = T_f, x \in (s,L)\ \forall t. \tag{7}$$

Because it is open to this type of analysis on the sub-domains, the Stefan Problem will be used in this paper for mathematical analysis.

B. Quasi-linear parabolic conservation of energy

Another PDE that can also be used to model the 1D slice, in fact the actual equation used in [2], is a generalized form of conservation of energy. Let the function $H(T)$ calculate the enthalpy—the thermodynamic internal energy—of the material at a temperature T, and $k(T)$ be the temperature-dependent conductivity. Then conservation of energy for heat diffusion can be written as:

$$\left(H(T(x,t))\right)_t = (k(T(x,t))T_x(x,t))_x, x \in (0,L). \tag{8}$$

The boundary conditions remain the same as in (4) and (5), and assumption (A1) can be similarly stated to ensure the problem is physically realistic.

It can be shown [4] that (8) and (1)-(2) are actually equivalent in a weak sense. Suppose k is constant, and H is defined as

$$H(T(x,t)) = \rho\left(c_p + L_f \cdot 1\left(T(x,t) - T_f\right)\right) \tag{9}$$

where ρ and c_p are the constant density and specific heat respectively, and $1(\cdot)$ is the unit step function. Then the weak forms of (8)-(9) and (1)-(2) under assumption (A1) are the same, with constants $a = k/\rho c_p$ and $b = k/\rho L_f$.

Of course, since (8) will then involve taking the derivative of a step function, it will not have a solution in the classical sense. This makes the equation more difficult to analyze mathematically. However, since it does not explicitly require a moving boundary at *s*, it may be numerically modelled on a fixed computational domain, with (9) slightly regularized. In addition, (9) may be generalized to model alloys, in which solidification occurs over a range of temperatures rather than a single step. This makes it more useful for simulation, as discussed in the next section.

C. Heat flux

For the example used in this paper, the boundary condition (5) is often modelled as [5]:

$$T_x(0,t) = q(t) = Au(t)^c(T(0,t) - T_\infty). \tag{10}$$

Here, $u(t)$ is the local spray water flux (flow rate through unit surface area) hitting the surface of the strand at time *t*, which is the main method of temperature control in a continuous caster. The term T_∞ is the temperature of the spray water, which is easily measured. The parameters *A* and *c* are the fitting parameters that depend on the design of the caster and the cooling sprays, and in general are different in different parts of the caster.

III. Previous Results

A. Continuous-in-time output feedback for the Stefan Problem

In [6] and [7], two possible output injection rules are proposed for the Stefan problem. Both assume the surface temperature can be measured throughout the caster, denote $y(t)$ as the measured surface temperature:

$$y(t) = T(0,t). \tag{11}$$

Both injection rules work by introducing $\hat{T}(x,t)$, $\hat{s}(t)$, the solution to a slightly-modified version of (1)-(5).

In [6], instead of the heat flux boundary condition (5), the Dirichlet boundary condition

$$\hat{T}(0,t) = y(t) = T(0,t) \tag{12}$$

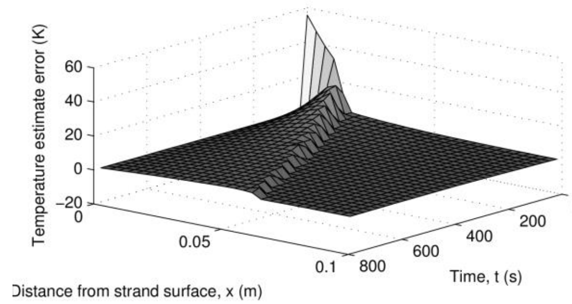
is used. This boundary condition simply forces the estimator to exactly match the measured surface temperature. This rule was proven to be stable, but not necessarily convergent. Convergence was seen in simulation, as shown in Figure 2(a).

In [7], instead of changing the boundary condition, output is injected through the Stefan condition. The equation (2) is changed to the estimate

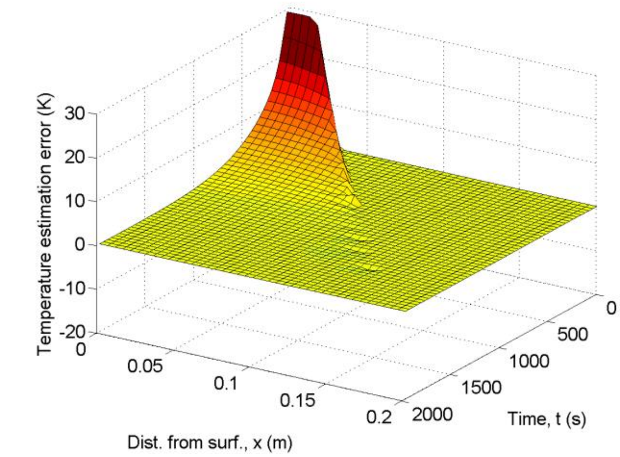
$$\hat{s}_t = -b\hat{T}_x\big|_{x=s^-}^{x=s^+} + L\left(\hat{T}(0,t) - y(t)\right). \tag{13}$$

This one is left as a conjecture and convergence or stability are still unproven. However, in simulation, it performs better than (12), showing what appears to be exponential convergence. An example simulation is shown in Figure 2(b).

These approaches have the advantage of being based on a fundamental mathematical analysis of the problem, even with no explicit proof existing at present. However, they assume more sensing than is typically available in the actual physical system, and without drastic improvements in sensing technology they will not provide a widely implementable solution for the problem.



(a) Estimation scheme (12) [6]



(b) Estimation scheme (13) [7]

Figure 2. Estimation error of Stefan problem estimators, using boundary sensing and two different estimation laws.

B. Open-loop "software sensor" estimator for quasi-linear conservation of energy

Within the steel industry, this problem of unreliable and sparse sensing has led to the widespread use of open-loop control for temperature in the caster. In fact, control systems in the industry are still open-loop in nature, albeit quite sophisticated: temperature "feedback" is obtained from real-time computational models instead of physical sensors [8-10]. For example, [10] uses the real-time mold cooling measurements in a state-of-the-art software sensor that gives small temperature error at the mold exit. Thus, although these "software sensor" models do take as inputs a wide range of measurements of casting conditions, the signal directly affected by the temperature of the strand itself in response to spray cooling is still unavailable to close the loop on the sprays through measurements taken in the spray zone. As a result, the estimate error, fairly small at the mold exit, grows as the distance from the latter increases.

IV. Discrete-in-Time Calibration

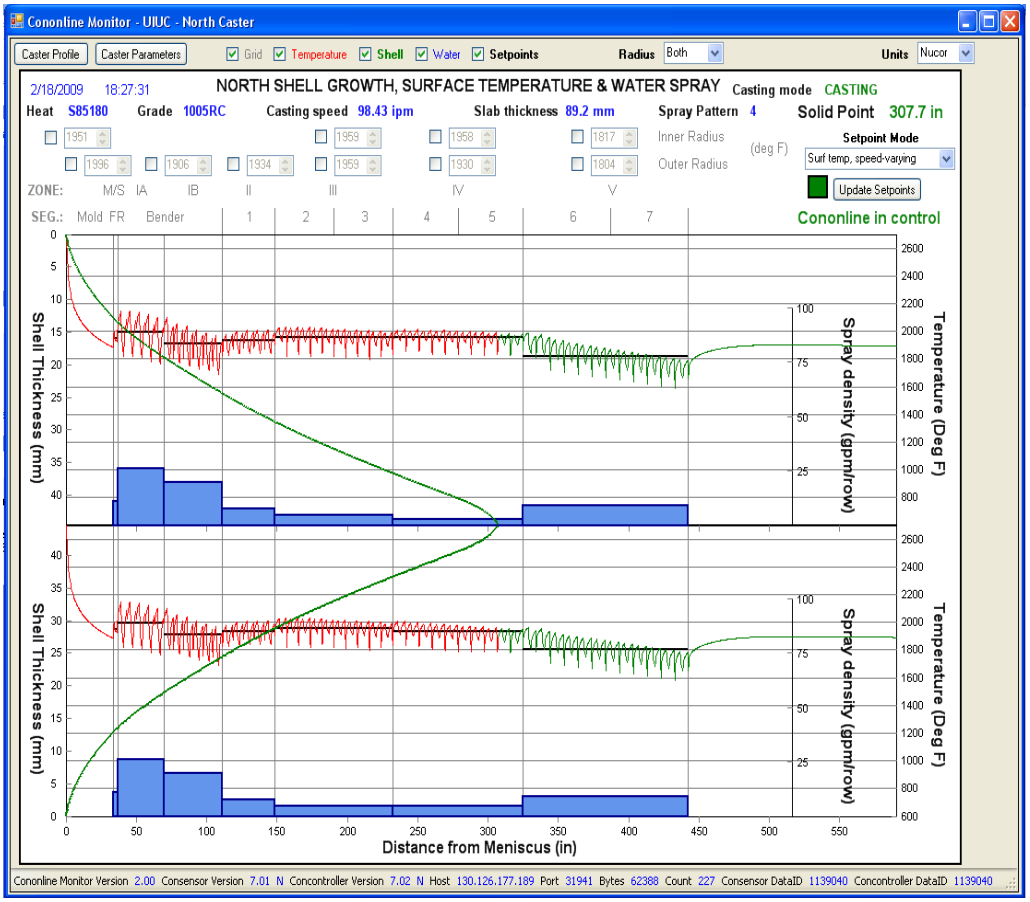


Figure 3. Human-machine interface from "software sensor" showing surface temperature and shell thickness output of computational model [11].

The purpose of this paper, then, is to start bridging the gap between the theoretical, but impractical approach of subsection III-A, and the practical but unproven approach of subsection III-B. The software sensors described in the previous section are open-loop in nature with respect to the spray zone. The only measurements available are not affected by the strand temperature - the output to be controlled through spray actuation. In fact, one of the problems the software sensor itself was specifically developed to solve is the lack of distributed temperature measurement in the strand. It is the nature of the problem that the only temperature measurements available, sparsely located pyrometers, are discrete-in-time and therefore do not lend themselves to standard estimation techniques.

However, one could attempt to "calibrate" the model - adjust its parameters to match sparse measurements - a common modeling problem. In general, this calibration problem assumes that the system dynamics are known except for a finite set of unknown parameters. In the present case of continuous casters, the least well-known parameters are those related to the boundary heat flux.

A. Calibration of the Stefan Problem

Although the Stefan Problem is nonlinear, it is still parabolic in most of the strand. In addition, the unknown parameters in the boundary heat flux (10) all affect the heat flux monotonically: increasing the parameter increases the heat flux. Therefore, we can apply the well-known properties of parabolic PDEs, in particular the maximum principle, to the error equations.

Lemma 1. *Let* $T_1(x,t)$, $s_1(t)$ *and* $T_2(x,t)$, $s_2(t)$ *be the solutions to* (1)-(5), *with the same initial conditions* $T_1(x,0) = T_2(x,0)$ *and* $s_1(0) = s_2(0)$ *that satisfy the initial condition assumption* (A1), *and have the same material properties a and b. Then, if the boundary heat fluxes satisfy*

$$q_1(t) > q_2(t) \; \forall t,$$

then the temperatures satisfy

$$T_1(x,t) < T_2(x,t) \; \forall t, \forall x \in [0, s_1).$$

Proof. The Proof is deleted due the page limitation and will be posted elsewhere.

An immediate consequence of this Lemma is that certain simple calibration problems must have a unique solution. For example, suppose the heat flux follows (9), assuming that only the parameter *A* is unknown. Under the physical assumptions on the other parameters and variables in (9), increasing *A* increases the heat flux. Then, by Lemma 1, if the pour temperature and a measurement at any single other point in the caster are available, the actual value of *A* can be found exactly.

Theorem 1. *Let* $T(x,t)$ *be the solution to the single-phase Stefan problem* (1)-(5) *under the assumption* (A1) *with the* *boundary condition* (10)*, where either A or c is unknown. If the initial condition* $T_0(x)$ *and a measurement* $T(x_{\text{measure}}, t_{\text{measure}})$, $x_{\text{measure}} \in [0, s_1)$, *are known, the unknown parameter can be found to an arbitrary accuracy.*

Proof. Applying Lemma 1 for a given initial condition T_0, since the heat flux (10) - and therefore the temperature gradient q - depends monotonically on A and c, the solution to the Stefan problem T will also depend monotonically on A and c. Therefore, there is a unique value of the unknown parameter that achieves a given measurement $T(x_{\text{measure}}, t_{\text{measure}})$. Furthermore, because of the monotonicity, it can be found, for example, by a simple binary search algorithm to any desired accuracy. □

B. Discussion and Simulation

Clearly, this result provides a strong conclusion, but requires strict conditions. The heat flux does have a monotonic dependence on parameters A and c, but the parameters may vary throughout the caster. So, the assumption of a single missing parameter is not very likely.

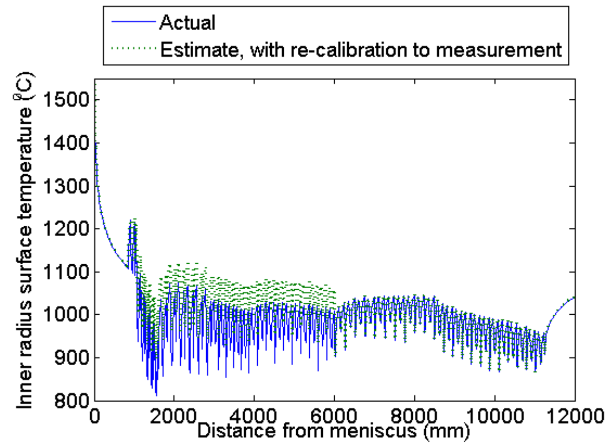
Extending this result to the more realistic PDE (8) is complicated as well. The equation is still parabolic under some simple realistic assumptions on the function $h(T)$. This means the maximum principle still applies to the PDE itself. The problem is the error PDE. Unlike the Stefan problem, the PDE for the error derived from (8) does not have a nice parabolic form of its own, and so the proof of Lemma 1 does not apply.

Nevertheless, the technique presented is seen below to provide a practical framework. Indeed, a simulation of (8)-(10), with working recalibration based on a single measurement is shown in Figure 4. Until the pyrometer is reached, at 6 m from the meniscus (indicated in Figure 1), the model assumes the value of A to be 1.57. In the actual system this value is 2. At 6 m from the top of the caster, a measurement of the surface temperature is obtained, for example from a pyrometer, dragging thermocouple, or infrared camera. A Newton search is used to adjust A to match the measurement. The derivative is calculated numerically using a simple finite-difference approximation. Within 2 iterations, the Newton search returns a value of 1.99 for A. The simulation is restarted with this value, and continues for the rest of the caster. As seen in Figure 4 (b), the surface temperature and the shell thickness estimation errors are practically eliminated after recalibration at 6000 mm distance from the meniscus.

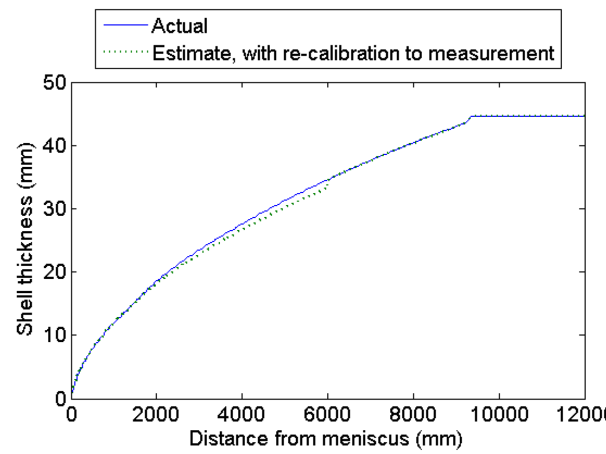
Figure 5 shows a software sensor of the type discussed in subsection III-B modified to include this parameter recalibration. The parameters A and c in equation (10) are updated as pyrometer measurements are taken for the slice. Figure 6 shows a slice-based control law which could conceivably use an estimator of the type in Figure 5 instead of direct measurements.

V. Conclusions and Future Work

In this paper a methodology for recalibration of online estimator through discrete-in-time temperature measurements to reduce the error caused by a single unknown parameter in the model is proposed. It is conjectured that changing the assumption from $T_0(x) = T_f, x \in (s_0, L)$ to $T_0(x) > T_f, x \in (s_0, L)$, i.e. to the two phase Stephen problem, the result could apply to measurements in $[s_1, L]$. This will be examined in future work.



(a) Surface temperature $T(0,t)$



(b) Shell thickness $s(t)$

Figure 4. Simulation of (8)-(9) showing mid-simulation calibration to match surface temperature measurement.

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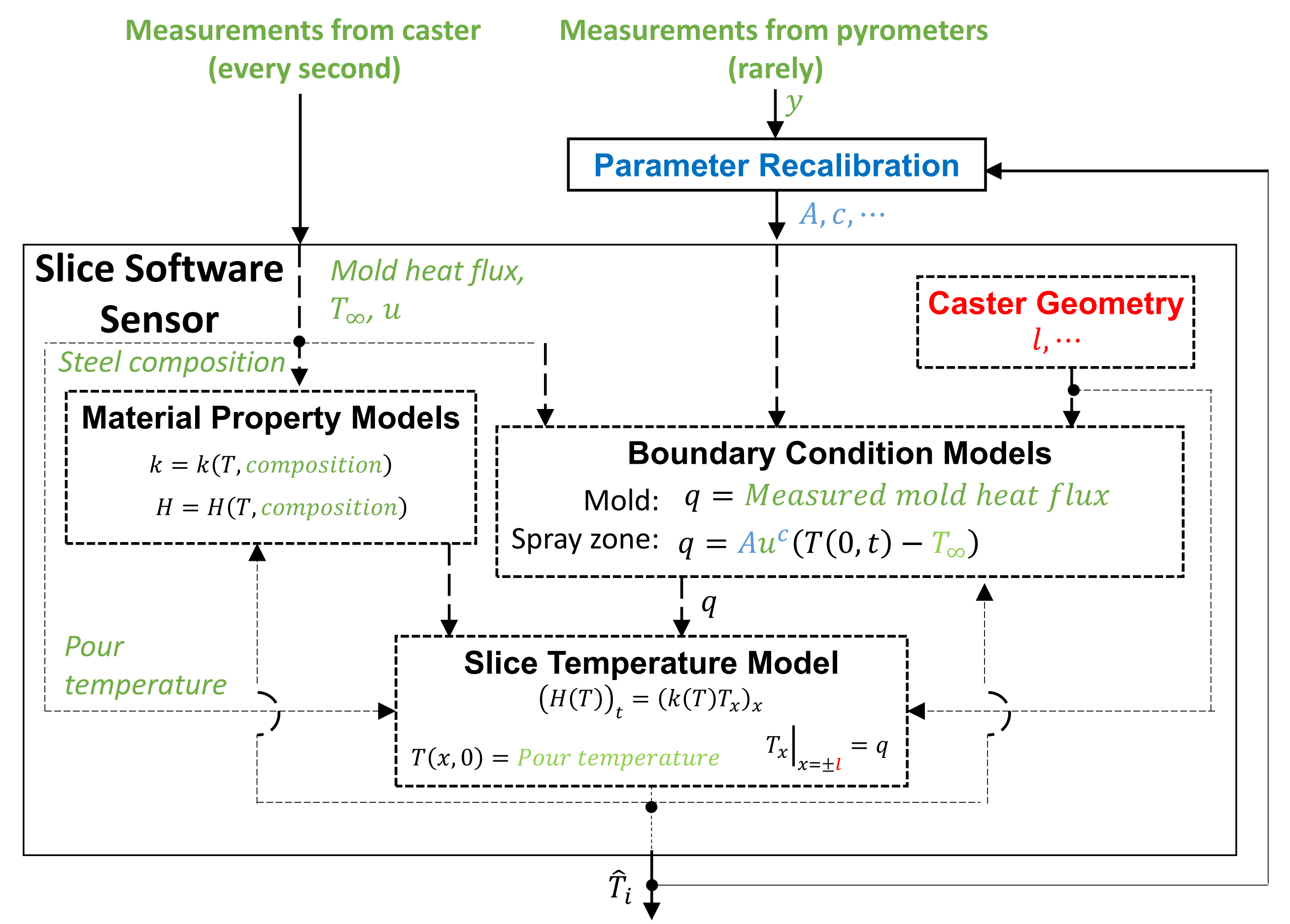

Figure 5. Modified block diagram for slice software sensor using real-time parameter recalibration from pyrometer measurements.

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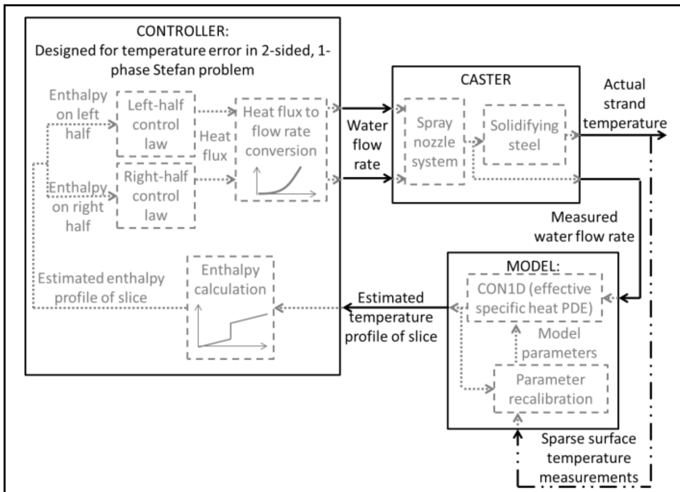

Figure 6. Block diagram illustrating configuration in which estimator with recalibration is intended to be used.