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# Rise of an argon bubble in liquid steel in the presence of a transverse magnetic field

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The rise of gaseous bubbles in viscous liquids is a fundamental problem in fluid physics, and it is also a common phenomenon in many industrial applications such as materials processing, food processing, and fusion reactor cooling. In this work, the motion of a single argon gas bubble rising in quiescent liquid steel under an external magnetic field is studied numerically using a Volume-of-Fluid method. To mitigate spurious velocities normally generated during numerical simulation of multiphase flows with large density differences, an improved algorithm for surface tension modeling, originally proposed by Wang and Tong [“Deformation and oscillations of a single gas bubble rising in a narrow vertical tube,” *Int. J. Therm. Sci.* **47**, 221–228 (2008)] is implemented, validated and used in the present computations. The governing equations are integrated by a second-order space and time accurate numerical scheme, and implemented on multiple Graphics Processing Units with high parallel efficiency. The motion and terminal velocities of the rising bubble under different magnetic fields are compared and a reduction in rise velocity is seen in cases with the magnetic field applied. The shape deformation and the path of the bubble are discussed. An elongation of the bubble along the field direction is seen, and the physics behind these phenomena is discussed. The wake structures behind the bubble are visualized and effects of the magnetic field on the wake structures are presented. A modified drag coefficient is obtained to include the additional resistance force caused by adding a transverse magnetic field. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4961561>]

## I. INTRODUCTION

Bubbly flows are encountered in a wide variety of industrial processes and in everyday life. In metallurgical processes, in order to mix and homogenize the metal, gas bubbles are injected at the bottom of a bulk liquid metal to stir the liquid metal.<sup>1</sup> In the process for continuous casting of steel, which is widely used for steel making, argon bubbles are commonly injected during the casting process. Understanding the motion of such argon bubbles is important as it has been shown that inclusions can be removed by bubble flotation.<sup>2</sup> In addition, in order to improve the product quality frequently an external magnetic field is applied to control the fluid motion and bubble behavior. Magnetic fields and bubbly flows are also encountered in the cooling system of TOKAMAK fusion reactors, where an external high magnetic field suppresses turbulence, resulting in low heat transfer rate and reduced cooling. Bubbles are injected to mix the liquid metal flow, reduce the suppressing effect of magnetic field, and thus increase the heat transfer rate.<sup>3,4</sup> In the past several decades, numerous<sup>5–13</sup> theoretical,<sup>7</sup> experimental,<sup>8–10</sup> and computational studies<sup>10–13</sup> have been carried out on the dynamics of a rising bubble in transparent liquids (such as water and oils). However, only a limited number studies have been reported on bubble motion in liquids when subjected to an

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external magnetic field. Since these liquids are usually metals and thus opaque, experiments to measure the bubble velocity have been difficult.

There are several important dimensionless numbers that govern the dynamics of the bubble rise. In most cases the Reynolds number is defined as  $Re_b = \rho_l \sqrt{gd} \mu_l^{-1}$  where  $d$  is the bubble diameter,  $\rho$  is density,  $\mu$  is viscosity,  $g$  is the standard acceleration due to gravity. The subscript  $l$  denotes liquid properties. The characteristic velocity used is  $\sqrt{gd}$  and sometimes it is also replaced by the bubble terminal rising velocity  $u_\tau$  which yields the terminal Reynolds number  $Re_\tau = \rho_l u_\tau d \mu_l^{-1}$ . The Eötvös number  $Eo = (\rho_l - \rho_g) g d^2 \gamma^{-1}$  (when  $\rho_l \gg \rho_g$ , the Eötvös number can be approximated by the Bond number  $Bo = \rho_l g d^2 \gamma^{-1}$ , where  $\gamma$  denotes the surface tension and subscript  $g$  denotes gas property) reflects the importance of surface tension force to gravitational force. It is used together with Morton number  $Mo = g \mu_l^4 (\rho_l - \rho_g) \rho_l^{-2} \gamma^{-3}$  to characterize the shape of bubbles. For a given pair of liquid and gas,  $Mo$  is fixed and  $Bo$  depends on the bubble size. The Archimedes number  $Ar = \rho_l |\rho_l - \rho_g| g d^3 \mu_l^{-2}$  gives the ratio of gravitational force to the viscous force. The confinement ratio is defined as  $C_r = W/d$ , where  $W$  is the width of the duct. The confinement ratio affects both the rise velocity and the path of the bubble.<sup>10</sup>  $N$  is the Stuart number (also known as magnetic interaction parameter) that describes the ratio of electromagnetic to inertial forces and  $N = Ha^2 Re_b^{-1}$ , where the Hartmann number  $Ha$  is defined as  $Ha = Bd \sqrt{(\sigma/\mu_l)}$ ,  $B$  is the strength of the magnetic field, and  $\sigma$  is the electrical conductivity.

Table I lists previous publications that have investigated the motion of single bubble rise in a conducting liquid with an external magnetic field. The terminal velocity of the bubble was measured by either the electrical triple probe method<sup>14</sup> or the Ultrasound Doppler Velocimetry method (UDV).<sup>15,16</sup>

Mori *et al.*<sup>14</sup> reported experimental results of a nitrogen bubble rising in mercury with an external magnetic field. With the use of an electrical triple probe, they showed that applying a magnetic field makes smaller bubbles ( $r = 1$  mm) rise slightly faster (due to the suppression of spiral motion by the magnetic field), while it causes larger bubbles ( $r = 3$  mm) to rise slower.

With the use of UDV, Zhang *et al.*<sup>15</sup> showed that a vertical external magnetic field considerably modifies the liquid velocity, and the oscillations in the bubble wake are suppressed. The electromagnetic field damped the liquid velocity and led to a more rectilinear bubble trajectory which is consistent with previous observations of Mori *et al.*<sup>14</sup> The results also showed that for bubbles of diameter less than 4.6 mm, an increase in the magnetic interaction parameter  $N$  leads to an increase of the drag coefficient. These experimental findings were numerically investigated by two groups of researchers<sup>17,18</sup> with different numerical approaches. Schwarz and Fröhlich<sup>17</sup> used an

TABLE I. Summary of studies of bubble rise under external magnetic field (Column 2: E—experimental, C—computational).

| Year | Author                                | E/C | Methods                 | Gas and liquid             | Field direction | Field strength                    | Bubble size (Eo or $d$ in mm)  |
|------|---------------------------------------|-----|-------------------------|----------------------------|-----------------|-----------------------------------|--------------------------------|
| 1997 | Mori <i>et al.</i> <sup>14</sup>      | E   | Electrical triple probe | N <sub>2</sub> and mercury | Horizontal      | $B = 0-1.5$ T                     | $d = 1.74-5.76$                |
| 2005 | Zhang <i>et al.</i> <sup>15</sup>     | E   | UDV                     | Ar and GaInSn              | Vertical        | $B = 0-0.3$ T<br>$N = 0-2$        | $d = 4.5-8.5$<br>Eo = 2.2–6.6  |
| 2014 | Schwarz <i>et al.</i> <sup>17</sup>   | C   | Lagrangian + IBM        | Ar and GaInSn              | Vertical        | $N = 0-2$                         | $d = 3-4.6$<br>Eo = 1.05–2.5   |
| 2014 | Zhang <i>et al.</i> <sup>18</sup>     | C   | VOF + AMR               | Ar and GaInSn              | Vertical        | $B = 0-0.5$ T<br>$N = 0, 4.3, 65$ | $d = 2.5-6.5$<br>Eo = 0.74–4.9 |
| 2010 | Shibasaki <i>et al.</i> <sup>19</sup> | C   | VOF + HS-MAC            | N/A                        | Vertical        | $Ha = 0, 25, 100, 200$            | Eo = 4.98                      |
|      |                                       |     |                         | N <sub>2</sub> and Mercury | Horizontal      | $B = 0, 0.5, 1, 1.5$ T            | $d = 5.6$                      |
| 2014 | Zhang <i>et al.</i> <sup>20</sup>     | C   | VOF + AMR               | Ar and GaInSn              | Horizontal      | $N = 0, 25, 100$                  | Eo = 24.5                      |
|      |                                       |     |                         | Ar and GaInSn              | Vertical        | $B = 0, 0.28, 0.87, 1.5$ T        | $d = 3-4.5$<br>Eo = 1.2–2.5    |

Eulerian-Lagrangian approach and immersed boundary method to simulate the motion of different size bubbles. For large bubbles, increasing the magnetic interaction parameter  $N$  caused the terminal bubble rise velocity to first increase but subsequently decrease with  $N$ . For small bubbles, an increase in the magnetic field caused a decrease in time-averaged bubble rise velocity. They also observed that with increasing  $N$ , both the bubble oscillation amplitude and characteristic frequency decrease. Zhang and Ni<sup>18</sup> used a Volume of Fluid (VOF) method and found that a moderate magnetic field increases the terminal velocity, while a stronger magnetic field leads to a reduced terminal velocity irrespective of the bubble size. This observation is different from experimental measurements.<sup>15</sup> The terminal velocity was also significantly different from measurements.<sup>15</sup> The wake structure behind the bubble was more regular and parallel when the magnetic field was applied.

Shibasaki *et al.*<sup>19</sup> also simulated the rise of a single bubble in a square duct subject to a vertical magnetic field using a VOF method. They observed that with an increase in the magnetic field strength, the shape of the bubble elongated in the rising direction. Low pressure regions in the upper and lower parts of the bubble were observed when the magnetic field was strong, and caused a vertical elongation of the bubble. They showed that with an increase in the strength of the magnetic field, the vortices behind the bubble tend to disappear due to the strong braking force in the transverse direction of the flow. The rise velocity of the bubble for  $Ha < 75$  is slightly larger than that without the magnetic field, but for  $Ha > 75$  the rise velocities decreased monotonically with increasing  $Ha$ .

Zhang and Ni<sup>20</sup> also studied the effect of a transverse magnetic field on the motion of a single gas bubble rising within an electrically conducting liquid. The simulations were done for Reynolds number  $Re_b = 125$ , Weber number  $We = 6.5$ , and Eötvös number  $Eo = 24.5$ . They observed that under a strong magnetic field ( $N = 25$  and  $100$ ) the bubble was greatly compressed in the direction parallel to the magnetic field. This result contradicts with previous work of small and moderate  $N$ , for which the bubble was elongated in the direction of magnetic field.<sup>18,19,21–23</sup> In the case of the transverse magnetic field, the terminal velocity was seen to decrease and become less oscillatory.

Summarizing the key publications in the literature, an external magnetic field is seen to make the bubble rise slower,<sup>14,15,17–20</sup> with a more rectilinear rising path,<sup>14,15,17</sup> with suppression and stabilization of the wake behind the bubble.<sup>15,17–19</sup> However to date, most of the works considered a vertical magnetic field<sup>15,17–19,22,23</sup> with only two studies with a horizontal magnetic field configuration.<sup>14,20</sup> The effect of a horizontal magnetic field on the bubble shape is still not well understood. Further, to our knowledge there are no studies that have examined the case of an argon bubble rising in liquid steel with a magnetic field. In this work, we study the effects of a transverse magnetic field on an argon bubble rising through a column of molten steel. We present a number of results from our simulations including the terminal velocity of the rising bubble, the wake structures, and the bubble path. The mechanisms of bubble elongation along field direction are discussed. From the results of this study, a modified drag coefficient is extracted to be used in practical calculations with large number of isolated bubbles (Lagrangian description).

This paper is organized first describing in Section II the governing equations, numerical method, and solution procedure. Section III provides results of validation and grid independence studies. Subsequently, results of several well-resolved calculations are presented and discussed in Section IV. Finally, Section V concludes the paper with a summary of the important results.

## II. GOVERNING EQUATIONS AND SOLUTION PROCEDURE

The problem considered in this study is a single Ar bubble rising in an initially quiescent column of liquid steel with a transverse external magnetic field (parallel to  $x$  axis) applied to the entire domain as shown in Fig. 1. The governing continuity and momentum equations are given by Eqs. (1) and (2),

$$\nabla \cdot (\rho \mathbf{u}) = 0, \quad (1)$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot [\mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)] + \mathbf{F}_L + \mathbf{F}_S + \rho \mathbf{g}, \quad (2)$$

where  $\mathbf{u}$  is the fluid's velocity,  $t$  is the time,  $\rho$  is the fluid's density,  $\mu$  is the fluid's dynamic viscosity,  $p$  is the total pressure, and  $\mathbf{g}$  is the gravity. The two source terms  $\mathbf{F}_L$  and  $\mathbf{F}_S$  represent Lorentz

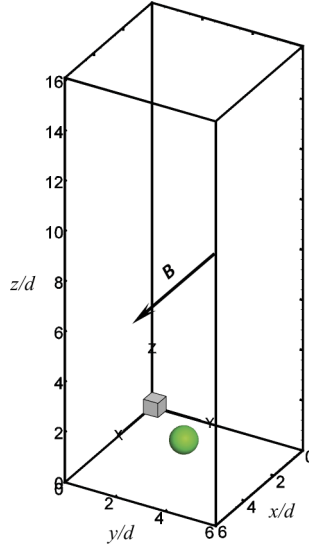


FIG. 1. Computational domain and the initial bubble location.

force and surface tension force, respectively. In this work, for an Ar bubble rising in liquid steel the magnetic Reynolds number  $Re_m = \mu_e \sigma d u_\tau$  is very small ( $\sim 10^{-3}$ ), where  $\mu_e$  is the magnetic permeability of free space. Hence, the induced magnetic field by the fluid motion is much smaller than the applied magnetic field, and therefore a quasi-static approximation can be used.<sup>24</sup> A potential method can then be used to compute the electric current and Lorentz force distribution. The Lorentz force  $\mathbf{F}_L$  is obtained by taking cross product of current density  $\mathbf{J}$  and external magnetic field  $\mathbf{B}$ , as shown in Eq. (3). The electrical current density  $\mathbf{J}$  can be computed through the Ohm's law as given by Eq. (4), and for a well conducting material the current conservation law is given by Eq. (5). Therefore, the electric potential  $\Phi$  satisfies Eq. (6). It is to be noted that the electrical conductivity of gas and liquid has different values (the conductivity of liquid steel is  $\sim 10^{20}$  larger than that of argon gas) and therefore the electrical conductivity  $\sigma$  in Eq. (6) cannot be canceled. With an insulated wall, the boundary condition for  $\Phi$  is  $\partial\Phi/\partial\mathbf{n} = 0$ ,

$$\mathbf{F}_L = \mathbf{J} \times \mathbf{B}, \quad (3)$$

$$\mathbf{J} = \sigma (-\nabla\Phi + \mathbf{u} \times \mathbf{B}), \quad (4)$$

$$\nabla \cdot \mathbf{J} = 0, \quad (5)$$

$$\nabla \cdot (\sigma \nabla\Phi) = \nabla \cdot [\sigma (\mathbf{u} \times \mathbf{B})]. \quad (6)$$

The surface tension force  $\mathbf{F}_S$  is evaluated using Eq. (7),

$$\mathbf{F}_S = \int_{\Gamma} \gamma \kappa \mathbf{n} \delta(\mathbf{x} - \mathbf{x}_f) d\mathbf{s}, \quad (7)$$

where  $\kappa$  denotes the mean interface curvature,  $\Gamma$  represents the interface,  $\mathbf{n}$  denotes the normal vector of the interface, and  $\delta$  is the Dirac delta function.  $\mathbf{x}$  and  $\mathbf{x}_f$  denote the coordinates of the cell and the interface, respectively. To capture a sharp interface and reduce spurious velocities, a Sharp Surface Force (SSF) method for modeling of the surface tension force is adapted. This SSF method, also known as Pressure Boundary Method (PBM) or Ghost Fluid Method (GFM), has been presented and discussed in detail elsewhere.<sup>25–28</sup> In this method, the surface tension in Eq. (7) is treated as a pressure gradient  $(-\nabla\tilde{p})$  which exactly balances the surface tension force  $\mathbf{F}_S$  generated due to presence of the interface. By considering that this new pressure field cannot generate velocity in a static case, another pressure Poisson equation can be obtained as given by Eq. (8) below,

$$\nabla \cdot \left( \frac{\nabla\tilde{p}}{\rho} \right) = F^{x+} - F^{x-} + F^{y+} - F^{y-} + F^{z+} - F^{z-}, \quad (8)$$

where the six terms on the RHS are scalars and defined as  $F^{x+} = -\left[\frac{\gamma\kappa}{\rho\Delta x^2}\right]_{(i+1/2,j,k)}$  and  $F^{x-} = -\left[\frac{\gamma\kappa}{\rho\Delta x^2}\right]_{(i-1/2,j,k)}$ , etc. The surface tension force at the interface is thus treated as a jump condition for calculating this new pressure field. The interface is tracked using the VOF method in which an evolution equation for the liquid volume fraction  $\alpha$ , given by Eq. (9), is solved,

$$\frac{\partial \alpha}{\partial t} + \mathbf{u} \cdot \nabla \alpha = 0. \quad (9)$$

A physical property  $\theta$  (i.e.,  $\rho$ ,  $\mu$  and  $\sigma$ ) at a given point in the domain is evaluated by linear interpolation as  $\theta = \alpha\theta_l + (1 - \alpha)\theta_g$ , where the subscript “l” denotes the property of the surrounding liquid and subscript “g” denotes the property of the gas phase. In the argon-steel system, the density ratio is of the order of  $\sim 10^4$ , viscosity ratio is  $\sim 10^2$ , and electrical conductivity ratio is  $\sim 10^{20}$ .

The above equations are converted to dimensionless form by using the following dimensionless variables:

$$\begin{aligned} \mathbf{x}^* &= \frac{\mathbf{x}}{d}; \quad \mathbf{u}^* = \frac{\mathbf{u}}{\sqrt{gd}}, \quad t^* = t\sqrt{\frac{g}{d}}, \quad \rho^* = \frac{\rho}{\rho_l}, \quad \mu^* = \frac{\mu}{\mu_l}, \\ p^* &= \frac{p}{\rho_l g d}, \quad \kappa^* = d\kappa, \quad \mathbf{g}^* = \frac{\mathbf{g}}{g}, \quad \mathbf{B}^* = \frac{\mathbf{B}}{B_0}, \quad \Phi^* = \frac{\Phi}{B_0 d \sqrt{gd}}, \quad \sigma^* = \frac{\sigma}{\sigma_l} \end{aligned} \quad (10)$$

the momentum equation is then re-written in a dimensionless form

$$\frac{\partial \rho^* \mathbf{u}^*}{\partial t^*} + \nabla \cdot (\rho^* \mathbf{u}^* \mathbf{u}^*) = -\nabla p^* + \frac{1}{\sqrt{Ar}} \nabla \cdot [\mu^* (\nabla \mathbf{u}^* + \nabla \mathbf{u}^{*T})] + \frac{Ha^2}{Re_b} \mathbf{F}_L^* + \frac{1}{Bo} \mathbf{F}_S^* + \rho^* \mathbf{g}^*. \quad (11)$$

A general purpose in-house code, CUFLOW,<sup>28-32</sup> for simulating laminar and turbulent flows was used to solve the above equations. The code employs Cartesian grids to integrate the three-dimensional unsteady incompressible Navier-Stokes equations. The continuity and momentum equations are solved using a fractional step method. Figure 2 shows the solution steps at each time step. In this study, an adaptive time step is used, with the time step  $\Delta t$  taken as  $\min(t_{\text{conv}}, t_{\text{visc}}, t_{\text{surf}})$ , where

$$\begin{aligned} t_{\text{conv}} &= C_{\text{CFL}} \left( \frac{u}{\Delta} + \frac{v}{\Delta} + \frac{w}{\Delta} \right)^{-1}, \\ t_{\text{visc}} &= C_{\text{CFL}} \frac{1}{6} \frac{\rho \Delta^2}{\mu}, \\ t_{\text{surf}} &= \frac{C_{\text{CFL}}}{2} \sqrt{\frac{\rho_l + \rho_g}{\pi} \cdot Eo \cdot \Delta^3}, \end{aligned} \quad (12)$$

where  $\Delta$  is the cell size and  $C_{\text{CFL}}$  is the Courant-Friedrichs-Lewy (CFL) condition. To be safe,  $C_{\text{CFL}} = 0.8$  is used in these simulations. Note that the effect of magnetic field on time step is not included because this study focuses on small  $N$  values. However, the effect of the magnetic field on limiting the time step must be considered if  $N$  is large.

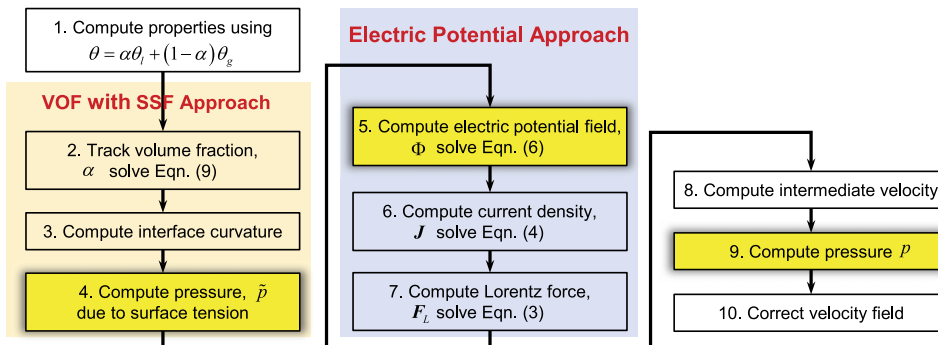


FIG. 2. A flow chart showing basic solution procedures.

TABLE II. Properties of air and water, argon, and steel.

|                             | Air                    | Water | Argon                  | Steel   |
|-----------------------------|------------------------|-------|------------------------|---------|
| $T$ (K)                     | 300                    |       | 1773                   |         |
| $\gamma$ (N/m)              | 0.0712                 |       | 1.2                    |         |
| $\rho$ (kg/m <sup>3</sup> ) | 1.17                   | 1000  | 0.56                   | 7 000   |
| $\mu$ (kg/(m s))            | $1.86 \times 10^{-5}$  | 0.001 | $7.42 \times 10^{-5}$  | 0.0063  |
| $\sigma$ (1/( $\Omega$ s))  | $1.00 \times 10^{-15}$ | 0.001 | $1.00 \times 10^{-15}$ | 714 000 |

Details of the solution algorithm and numerical implementation are available elsewhere.<sup>28–32</sup> The most computationally intensive parts are the solutions of the three Poisson equations (pressure-Poisson equation, electrical-Poisson equation, and the surface tension related Poisson equation). In this work, the three Poisson equations are solved efficiently by a V-cycle multigrid method, and red-black Successive Over Relaxation (SOR) with over-relaxation parameter of 1.6.

### III. CODE VALIDATION AND GRID INDEPENDENCE STUDY

The solver has been previously validated in a number of problems such as the lid-driven cavity flow of a Newtonian fluid with and without a magnetic field against published results.<sup>32</sup> The VOF with SSF implementations were also validated through comparing predicted bubble rise velocity and shape at high Morton numbers.<sup>28</sup> To further validate the algorithm in a low Morton number and high density ratio regime, two more validations are performed: (1) prediction of the shape and rise velocities of single air bubbles rising in water (with 6 different bubble sizes); and (2) the rise velocity of an argon bubble rising in GaInSn. An additional validation is performed for a N<sub>2</sub> bubble rising in mercury with a horizontal magnetic field.

#### A. Validation 1—rise of an air bubble in water

Several simulations of air bubbles of different sizes rising in a container with quiescent water were first carried and compared with previous experimental and computational works. The properties of air and water used in these validations are listed in Table II. The important dimensionless numbers are provided in Table III. It is important to note that both systems (Ar-steel and air-water) are in the low Eötvös number and low Morton number regime, and therefore yield similar bubble Reynolds numbers  $Re_b$  (around 550 for a 3 mm bubble). In this study, the liquid steel is modeled as a Newtonian fluid which is usually the case when the superheat is greater than 20 K.<sup>33</sup>

Six validation simulations were conducted for air bubbles rising in water. The selected air bubbles have diameters ranging from 1 mm to 7 mm which correspond to Eötvös numbers from 0.1 to 6.7. In each case, a single bubble was released from the bottom of a container ( $4d$  in width and  $10d$  in bubble rising direction). A grid containing  $128 \times 128 \times 384$  finite volume cells was used in each validation study. The shape of bubble was initialized to be a sphere with zero velocity and was placed at  $1d$  above the bottom wall of the container. For bubbles with different Eötvös number, the predicted terminal bubble Reynolds numbers  $Re_\tau$  are plotted in Fig. 3. The bubble shape and iso-Morton number lines are obtained from previous studies.<sup>34,35</sup> It can be seen that all the points lie between the two lines of  $Mo = 10^{-10}$  and  $10^{-12}$  (if the  $Mo = 10^{-12}$  is extended) and those points would be roughly close to the line of  $Mo = 10^{-11}$  which is the Morton number for air and water. It

TABLE III. Dimensionless numbers of air-water and argon-steel ( $d = 3$  mm).

|           | Eo   | Mo                    | $Re_b$ | $\rho_l/\rho_g$     | $\mu_l/\mu_g$ | $\sigma_l/\sigma_g$  |
|-----------|------|-----------------------|--------|---------------------|---------------|----------------------|
| Air-water | 1.24 | $2.7 \times 10^{-11}$ | 514.4  | $8.547 \times 10^2$ | 53.8          | $1.0 \times 10^{12}$ |
| Ar-steel  | 0.51 | $1.3 \times 10^{-12}$ | 571.5  | $1.250 \times 10^4$ | 84.3          | $7.4 \times 10^{20}$ |

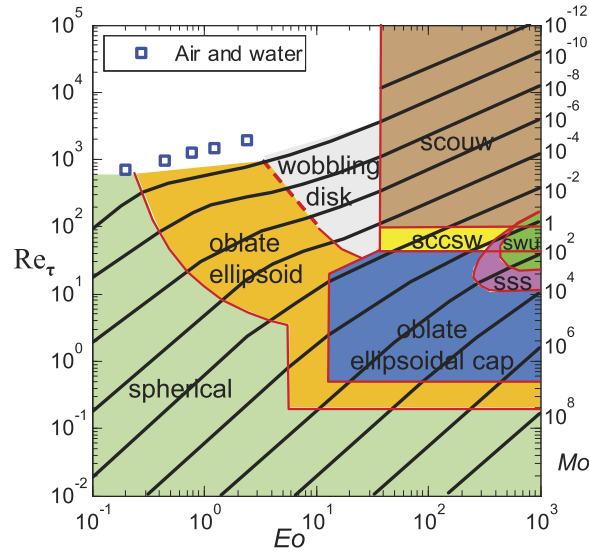


FIG. 3.  $Eo$  vs. predicted  $Re_\tau$  on the Grace diagram. Bubble shape: scouw—spherical cap with open unsteady wake; sccsw—spherical cap with closed steady wake; swu—skirt with wavy unsteady skirt; sss—skirt with smooth steady skirt.

is important to mention that the observed bubble shapes also agree well with the Grace diagram: the 1 mm bubble was found to be spherical in shape, and the 7 mm bubble was seen to oscillate between an oblate spherical and a “disk” shape. All other bubbles were found to be in oblate spherical shape.

The terminal velocity of the bubble is plotted in Fig. 4(a) together with some of the available data. In Fig. 4(a) the upper dashed line is the terminal velocity of an air bubble rising in distilled water<sup>5</sup> based on previous experimental work,<sup>9</sup> while the bottom line is the terminal velocity of an air bubble rising in surfactant added water. One can see that with surfactants added, the terminal rise velocity is reduced by as much as ~60%. It is important to mention that these two lines merge for bubbles of small size because even a small amount of surfactant contained in the distilled water can prevent circulation inside of the bubble,<sup>5</sup> leading to higher drag and lower terminal velocity. The triangles are UDV measurements.<sup>15</sup> The predicted terminal velocities using our current code are close to the 3D Front Tracking Method (FTM) simulation results reported by Dijkhuizen *et al.*<sup>11</sup> It can be seen from the figure that both numerical methods predict slightly higher velocities than

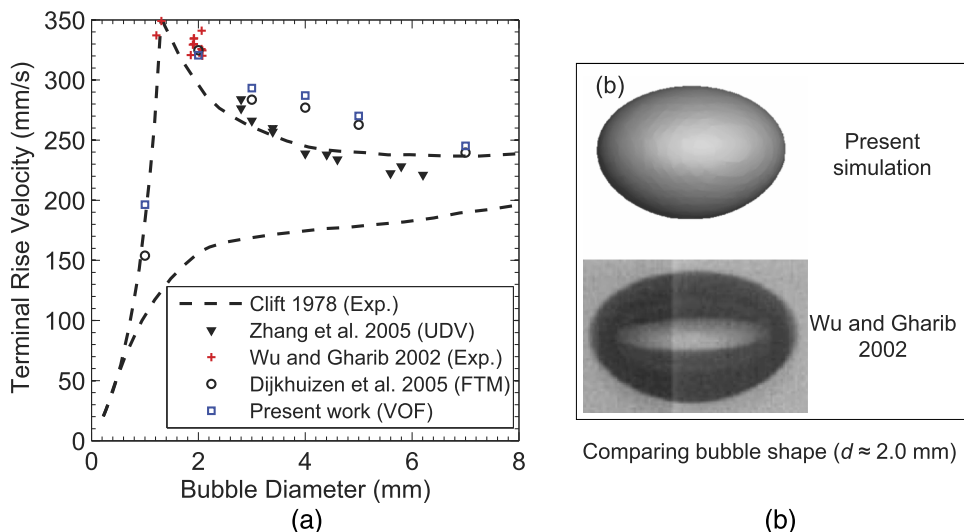


FIG. 4. (a) Comparison of predicted terminal velocities of bubbles of different size. (b) Predicted shape of a 2 mm bubble after the bubble travels ~6.7 mm from release.

observed in UDV experiments. The smaller terminal velocities in the experiments may be due to the small amounts of contamination still contained in distilled water which affects the circulation inside the bubble and breaks the “boundary condition” between the air bubble and the surrounding water. This leads to a higher drag and a lower terminal velocity. To minimize the impurities, Wu and Gharib<sup>36</sup> performed experiments of a single air bubble (up to  $\sim 2$  mm) rising in water using filtered air ( $0.2\ \mu\text{m}$  air filters) and clean water which is taken from a deionized water source that has been pretreated by a water purification system, then distilled by an auto-distiller, and filtered by a 3-module filtration system. Bubbles were slowly injected from a capillary of diameter  $0.0267$  cm. Their results are shown as red “+” markers in Fig. 4(a) which match very well with our predictions. Fig. 4(b) compares the shape of a  $2.0$  mm bubble from the experiments of Wu and Gharib<sup>36</sup> after it travels  $\sim 6.7$  mm and present computations. Both the experiment and the presents simulation show similar ellipsoidal bubble shapes. Dijkhuizen *et al.*<sup>37</sup> have also mentioned that their results match closely with the experimental work of Veldhuis<sup>38</sup> who studied the rise of air bubbles in “ultra-pure water.” In these experiments when the bubble size is large ( $d > 1.3$  mm), a zig-zag or helical motion was reported<sup>5,8</sup> which is also seen in our simulation for  $d \geq 2$  mm bubbles. For  $d = 1$  mm, the rectilinear bubble path seen in most experiments<sup>5,8</sup> is predicted in our simulation. The terminal velocity matches very well in this case. However, it is important to point out that the predicted terminal velocity even for the  $1$  mm bubble is still slightly higher than that observed in the experiment due to the fact that “pure” water used in numerical simulations yields a free mobile interface, which leads to lower drag and higher rising velocity.

## B. Validation 2—argon bubble rise in GaInSn

A second validation test and grid independence study was next done by simulating an argon bubble of  $d = 2.5$  mm rising in quiescent GaInSn container ( $C_r = 2$ ) with a height of  $8d$ . The bubble is initialized as a sphere and placed at the center of the duct and  $1d$  above the bottom wall. Three simulations using 24, 32, and 48 cells across a single bubble were performed. The properties<sup>15,18</sup> of GaInSn and Ar gas at 293 K used in these three simulations are given in Table IV. The dimensionless numbers for these simulations are  $Bo = 0.74$ ,  $Mo = 2.38 \times 10^{-13}$ , and  $Re_b = 1140$ . The bubble rise history curves for the three grid resolutions are plotted in Fig. 5(a). As the grid is refined, the rise velocity increases slightly. Comparing these curves, we infer that  $\Delta x = d/32$  and  $\Delta x = d/48$  are in close agreement. Refining the mesh from  $\Delta x = d/32$  to  $\Delta x = d/48$  changes the predicted terminal velocity only by 1.6%. Therefore, we have used  $\Delta x = d/32$  in our simulations.

In Fig. 5(b) the predicted rise velocity of this  $2.5$  mm bubble is plotted against the UDV measured terminal velocity,<sup>15</sup> other numerical predictions,<sup>18</sup> and a theoretical prediction.<sup>7</sup> The line in Fig. 5(b) is the terminal velocity suggested by Mendelson<sup>7</sup> who used an analogy between waves and bubbles, and treated the large bubbles as an interfacial disturbance. By using  $\pi d$  as the wave length, the terminal velocity (equal to the traveling speed of the wave) can be obtained as

$$u_\tau = \sqrt{2\gamma d^{-1}\rho^{-1} + 2gd}. \quad (13)$$

It has been shown that this wave analogy predicts rising velocity close to experimental data in the air-water system for bubble diameter greater than  $1.3$  mm.<sup>5</sup> In Fig. 5(b), the measured terminal velocities in the UDV experiments are always lower than those predicted by the theory, which may due to the impurities (oxides) in the GaInSn that lead to a lower surface tension<sup>15</sup> and less

TABLE IV. Physical properties of argon and GaInSn at  $T \approx 300$  K.

|                             | Argon                  | GaInSn                 |
|-----------------------------|------------------------|------------------------|
| $\gamma$ (N/m)              |                        | 0.533                  |
| $\rho$ (kg/m <sup>3</sup> ) | 1.654                  | $6.361 \times 10^3$    |
| $\mu$ (kg/(m s))            | $1.176 \times 10^{-5}$ | $2.200 \times 10^{-3}$ |
| $\sigma$ (1/( $\Omega$ s))  | $1.0 \times 10^{-15}$  | $3.27 \times 10^6$     |

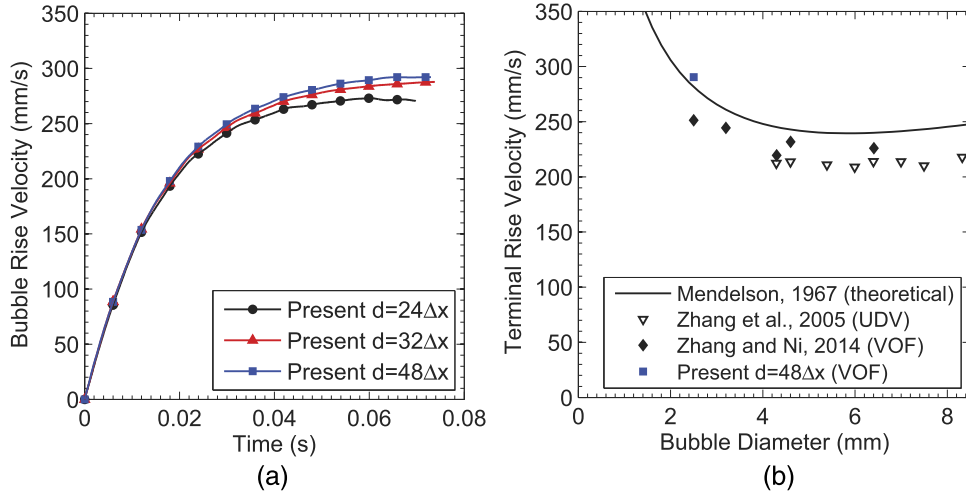


FIG. 5. (a) History of rise velocity of 2.5 mm argon bubble rising in GaInSn. (b) Comparison of predicted terminal velocity and available data for argon bubbles in GaInSn.

slippery interface between the bubble and the surrounding liquid. Thus the drag is increased and the rise velocity is reduced. Our numerical result is closer to that reported by Zhang and Ni<sup>18</sup> and the theoretical solution.<sup>7</sup>

### C. Validation 3—N<sub>2</sub> bubble rise in mercury with horizontal magnetic field

To further validate the multi-physics code, a bubble rising in an electrically conducting fluid with a magnetic field is considered. Although measurements<sup>15</sup> are available for a single argon bubble rising in GaInSn with a vertical magnetic field, the measured rise velocity is influenced by contaminations (oxides) in GaInSn. Thus, a simulation of a N<sub>2</sub> bubble ( $d = 5.6$  mm) rising in mercury with a horizontal magnetic field of 0.5 T was conducted on a grid consisting of  $128 \times 128 \times 384$  cells. The computational domain is the same as that shown in Figure 1. The physical properties of mercury and N<sub>2</sub> are given in Table V. Figure 6 compares the predicted rise velocity with experimental results<sup>14</sup> and previous numerical predictions.<sup>20</sup> The results show that with a horizontal magnetic field of 0.5 T, a  $d = 5.6$  mm N<sub>2</sub> bubble in mercury rises at  $\sim 162$  mm/s, which is very close to the measured<sup>14</sup> rise velocity of  $\sim 168$  mm/s and the previous numerical prediction<sup>20</sup> of  $\sim 175$  mm/s. The measured velocity reported by Mori *et al.*<sup>14</sup> is a time-averaged velocity after the bubble passes the initial rise stage. Note that in the case without the magnetic field, the predicted mean rise velocity of the N<sub>2</sub> bubble is 208 mm/s with large shape and velocity oscillations. The predicted mean rise velocity agrees well with the analytical prediction of 199 mm/s (Mendelson equation<sup>7</sup> given by Equation (13)) and also matches with that reported by Zhang and Ni.<sup>20</sup>

## IV. RESULTS AND DISCUSSIONS

We now present results of several numerical studies in the geometry shown in Fig. 1. The computational domain has a cross section of  $6d \times 6d$  and a length of  $16d$ . A grid of  $192 \times 192 \times 512$  ( $\sim 19 \times 10^6$ ) cells in the  $x$ ,  $y$ , and  $z$  direction was used to discretize the governing equations. A

TABLE V. Physical properties of N<sub>2</sub> and mercury at  $T \approx 300$  K.

|                             | N <sub>2</sub>        | Mercury               |
|-----------------------------|-----------------------|-----------------------|
| $\gamma$ (N/m)              |                       | 0.4865                |
| $\rho$ (kg/m <sup>3</sup> ) | 1.17                  | $1.35 \times 10^4$    |
| $\mu$ (kg/(m s))            | $1.77 \times 10^{-5}$ | $1.50 \times 10^{-3}$ |
| $\sigma$ (1/(Ω s))          | $1.0 \times 10^{-15}$ | $1.02 \times 10^6$    |

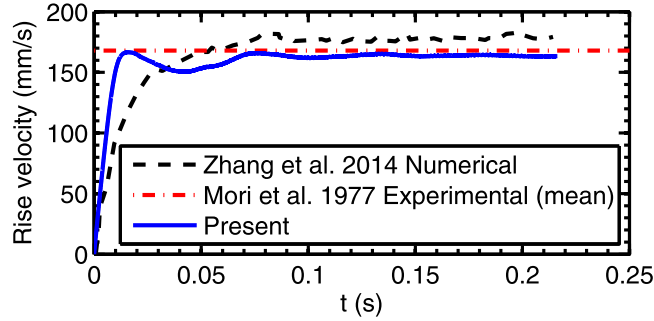


FIG. 6. Rise velocity of a  $d = 5.6$  mm ( $Eo = 8.5$ )  $N_2$  bubble rise in mercury subject to a horizontal magnetic field of 0.5 T ( $N = 0.45$ ).

spherical argon bubble of diameter  $d$  was initially placed at the center and  $0.5d$  above the bottom wall of the container. A uniform magnetic field of strength  $B$  was applied in the horizontal direction ( $x$ ) and kept constant throughout the simulation. The dynamics of the bubble, surrounding fluid, and their interaction with the Lorentz force during the rise process are studied. The physical properties of the hot argon gas and the liquid steel are provided in Table II. Table VI lists parameters of 9 simulations with different bubble sizes and magnetic field strengths considered in this study. The cases investigated have a  $Eo$  of 0.51, 1.43, or 2.80,  $Re_b$  was varied from 572 to 2037, and  $Ha$  was increased from 0 to 37.26. The results are presented in the following structure: Section IV A focuses on the effect of the magnetic field on bubble rise velocity, deformation and oscillation frequency. Section IV B analyzes the velocity field and the Lorentz force distribution. The interaction between the velocity field, bubble shape, and the Lorentz force is discussed; Section IV C presents the wake structures behind the rising bubble; Section IV D studies the effect of the magnetic field on the drag coefficient.

### A. Bubble rise velocity and deformation

The dimensionless rise velocities  $W^*$  versus dimensionless time  $t^* = t\sqrt{g/d}$  for all cases are first shown in Fig. 7. The rise velocity  $W^*$  is computed as the volume average of the vertical velocity  $w^* = w/\sqrt{gd}$  of the bubble. For 3 mm bubbles, Fig. 7(a) shows that for all cases the rise velocity curves are smooth and non-oscillatory. In the initial stages ( $t^* < 0.5$ ) the three rise curves are almost coincident. Without the magnetic field, the rise velocity of a 3 mm bubble first increases to a value of 2.5 and then starts to decrease slightly. This decreasing trend is caused by the inclined motion of the bubble, which will be discussed later. After applying a transverse magnetic field of  $B = 0.2$  T, the rise velocity is reduced by  $\sim 4\%$  from the peak value of the zero magnetic field case. In the case with zero magnetic field, the rise velocity ( $w^*$ ) reduces after  $t^* = 6$  due to a non-rectilinear motion of the bubble, however, its speed remains the highest. When a magnetic field of 0.5 T is applied, the rise velocity is seen to decrease to 1.83, a 24% reduction from the maximum rise velocity without any magnetic field. Figure 7(b) shows the rise velocities for 5 mm bubbles. Without a magnetic field, the rise velocity of a 5 mm bubble shows similar trends as that of the 3 mm bubble. The rise velocity increases to a maximum value 1.72 and then starts to decrease. Differing from the 3 mm case, the rise velocity of the 5 mm bubble slightly oscillates. After applying a

TABLE VI. List of simulations (Ar bubble rising in liquid steel, 1773 K,  $Mo = 1.3 \times 10^{-12}$ ).

| $d$ (mm) | $Eo$ | $Re_b$ | No. <sup>a</sup> | $B$ (T) | $Ha$ | $N$ | No. <sup>a</sup> | $B$ (T) | $Ha$  | $N$  | No. <sup>a</sup> | $B$ (T) | $Ha$  | $N$  |
|----------|------|--------|------------------|---------|------|-----|------------------|---------|-------|------|------------------|---------|-------|------|
| 3        | 0.51 | 572    | 1                | 0.0     | 0    | 0   | 2                | 0.20    | 6.39  | 0.07 | 3                | 0.50    | 15.97 | 0.45 |
| 5        | 1.43 | 1230   | 4                | 0.0     | 0    | 0   | 5                | 0.22    | 11.58 | 0.11 | 6                | 0.54    | 28.96 | 0.68 |
| 7        | 2.80 | 2037   | 7                | 0.0     | 0    | 0   | 8                | 0.20    | 14.90 | 0.11 | 9                | 0.50    | 37.26 | 0.68 |

<sup>a</sup>No. is the simulation number, and the nine simulations are numbered from 1 to 9.

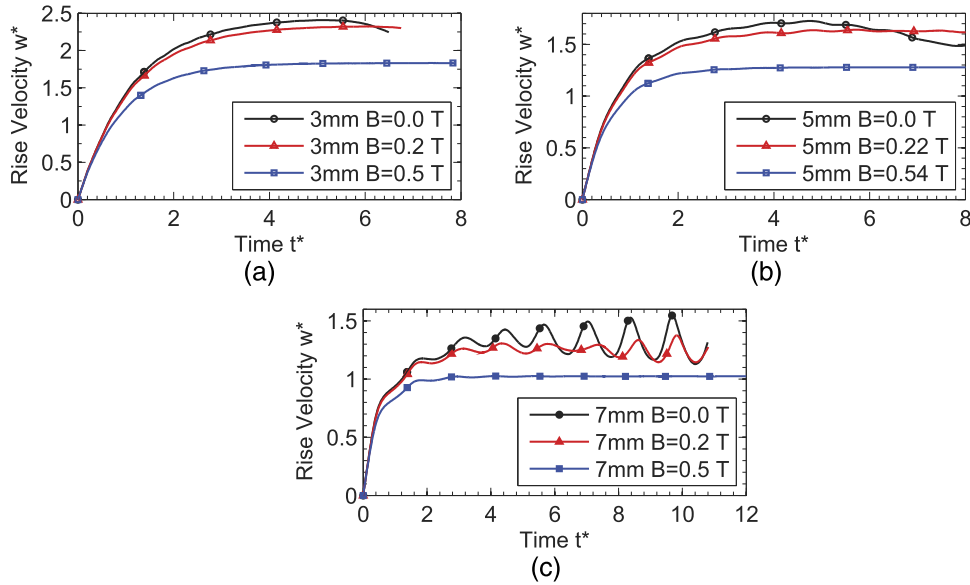


FIG. 7. (a) Rise velocity of a 3 mm argon bubble. (b) Rise velocity of a 5 mm argon bubble. (c) Rise velocity of a 7 mm argon bubble.

magnetic field of  $B = 0.22$  T ( $N = 0.11$ ), the rise velocity is slightly reduced but still oscillatory. The oscillations completely disappear after an increase in the magnetic field to  $0.5$  T ( $N = 0.68$ ), with the rise velocity being reduced to  $\sim 1.27$  which is 25% smaller than that without the magnetic field. Figure 7(c) shows the rise velocities for 7 mm bubbles. It is seen that the rise velocity for the 7 mm bubble without the magnetic field is oscillatory after the initial rise ( $t^* > 1.0$ ). These velocity oscillations are a result of the oscillations in bubble shape, as will be explained later. The peaks and valleys of the rise velocity correspond to smaller and larger drag on the bubble, as the bubble shape expands and contracts along different axes (the bubble volume is ensured to remain constant). With no magnetic field, the bubble oscillations persist until the end of the duct, reaching a periodic oscillatory motion. When a magnetic field of  $0.2$  T is applied, the amplitude of oscillations is considerably dampened. Until about  $t^* = 1.8$ , the two rise velocity curves are nearly the same, indicating nearly the same drag on the bubble and nearly same bubble deformation. However, at  $t^* = 1.8$ , the rise velocity curves depart and the oscillations are dampened by the magnetic field. The amplitude decreases as a result of different bubble shapes (to be presented below) and drag. Further increase of the magnetic field to  $0.5$  T completely damps the oscillation, resulting in a steady rise velocity. Under the strong magnetic field of  $0.5$  T, the rise velocity is seen to be reduced by  $\sim 25\%$ , from the no magnetic field average value. Here we focus on the dynamics of the 3 mm and 7 mm bubbles, because their shapes are in different regimes.

### 1. Dynamics of the 3 mm bubble

Figures 8–12 show top and side views of the 3 mm bubble at different times for the three magnetic field strengths. With no magnetic field, the top views of the bubble in Fig. 8 show that at  $t^* = 2$  the maximum bubble cross-sectional diameter is  $\sim 1.18d$ . After that, the maximum cross-sectional diameter remains the same. For  $t^* < 4$  the bubble is almost in a rectilinear motion and remains at the center of the duct. The rise velocity plot shows that at  $t^* \approx 5$  the rise velocity reaches its maximum and at that time it is already biased from the centerline of the duct. Subsequently, the acceleration of the bubble becomes smaller and the biased motion of the bubble causes a reduction in vertical rise velocity. At  $t^* = 6$  the top view shows that the bubble has already tilted and it is moving toward the southeast corner of the duct. At  $t^* = 6.5$  the bubble tilts further away from the centerline of the duct, further lowering the vertical component of the rise velocity.

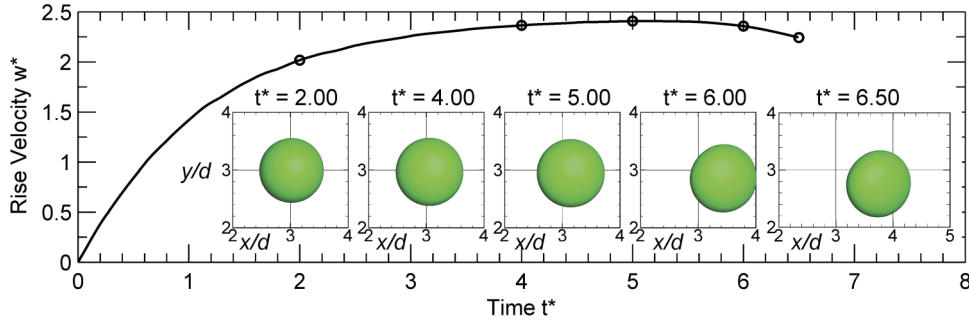


FIG. 8. Bubble rise velocity and the associated top-views of the 3 mm bubble at different times (indicated as black dots on the line plot) for  $B = 0$  T.

The side and front views of the bubble for  $B = 0$  T at different times are shown in Fig. 9. These side views clearly show that at later times, the bubble deforms to an ellipsoidal shape. A close examination of the surface curvature shows that the front of the bubble is flatter than the back of the bubble. This may be due to the flow impinging on the top and the pressure on the top surface being higher. The bubble is seen to be tilted by around  $25^\circ$  and is biased from the center of the duct. The dashed lines in the figure are approximate tangents to the top (outer surface) of the bubble. The average rise velocity can be estimated also using the  $z$  dimension of the dashed line divided by the time interval (0.75). By using this graphical method the computed average bubble rise velocity for the 3 mm bubble for  $B = 0$  T is 2.46. This estimated average rise velocity agrees well with that predicted using the volume integral method that was presented in Fig. 8.

The corresponding rise velocity and top views of the 3 mm bubble with the 0.2 T transverse magnetic field are shown in Fig. 10. With the external magnetic field, the rectilinear motion of the bubble lasts longer and it is seen that at  $t^* = 5$  the bubble still remains at the centerline of the duct. The maximum cross sectional diameter of the bubble is seen to be  $\sim 1.18d$ . Top views of the bubble at  $t^* = 6$  and 6.5 reveal that the bubble moves upwards toward the northeast corner of the duct. Although the horizontal motion of the bubble is seen for both  $B = 0.2$  T and  $B = 0$  T cases, the bubble is not tilted significantly. The rise velocity only slightly decreases after  $t^* = 6$ , which is again caused by the transverse motion of the bubble.

Figure 11 shows the side and front views of the 3 mm bubble for  $B = 0.2$  T at different times. Comparing the bubble shape observed, it can be seen that the magnetic field of 0.2 T does not significantly modify the shape of the small bubble ( $d = 3$  mm) and no preferential elongation in  $x$  or  $y$  direction is seen. The bubble is seen to rise along the center region of the container and it does not tilt as observed in Fig. 9 when no magnetic field is applied. At  $t^* = 5.75$  the top of the bubble reaches  $z = 12.2d$  which is  $0.4d$  below the location observed in the case with  $B = 0$ . The dashed lines in the figure are again approximate tangents to the top (outer surface) of the bubble, and the estimated average rise velocity with the graphical method is 2.4 which is slightly smaller than that seen without magnetic field.

Further increase of the magnetic field to 0.5 T makes the bubble shape more stable as the bubble remains at the center of the duct cross section (Fig. 12) at  $t^* = 7$ . No significant rotation of the bubble is observed. The side views of the bubble reveal that the bubble is ellipsoidal and is slightly elongated in the magnetic field direction ( $x$  direction). This is a result of the distribution of

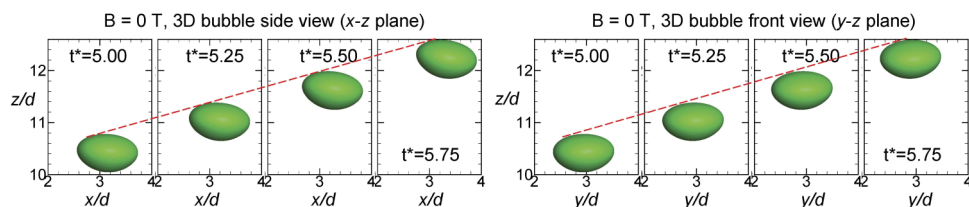


FIG. 9. Bubble shape at different times  $t^*$ ,  $d = 3$  mm,  $B = 0$  T.

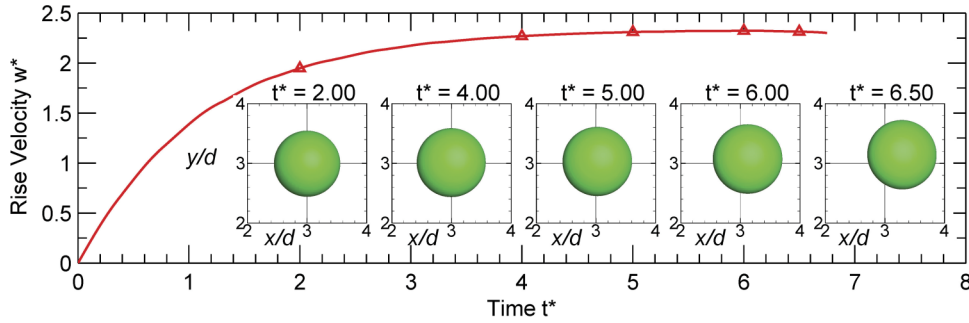


FIG. 10. Bubble rise velocity and the associated top-view of the 3 mm bubble at different times (indicated as red triangle on the line plot) with  $B = 0.2$  T.

the Lorentz force which will be discussed in Section IV B. The aspect ratio (the largest diameter along  $z$  divided by the largest diameter along  $x$ ) of the bubble is calculated to be  $\sim 0.72$ . The top of the bubble reaches  $z = 12.7d$  at  $t^* = 7$ , whereas when there is no magnetic field the bubble arrives at the same position much earlier at  $t^* = \sim 5.75$ .

## 2. Dynamics of the 7 mm bubble

Figure 7(c) showed a large oscillation in the rise velocity of the 7 mm bubble, implying large deformations of the bubble and oscillatory drag. However, bubble velocities in the initial stage are nearly the same with and without the magnetic field. This was also observed in a previous study.<sup>18</sup> Figure 13 shows the bubble shape with no magnetic field, in the time interval of  $t^* = 0$ –1.8. Figure 13 shows that the bubble undergoes a significant deformation between  $t^* = 0$  and 1.25. The bubble first changes from a sphere to a “mushroom-head-like” shape at  $t^* = 0.75$  and then deforms into a squeezed (in  $z$  direction) ellipsoidal shape at  $t^* = 1.25$ . During this time interval, the bottom half of the bubble moves up but the front of the bubble does not rise as much. This is due to the larger pressure on the top surface and the inability of the bubble to displace the liquid in front. It is important to note that here the rise velocity predicted by the graphical method (dashed line) will be smaller (if we draw the line at top) or larger (if we draw the line at bottom) than the integration method depending on where the line is drawn. The rise velocity curves with and without the magnetic field during the early stage ( $t^* < 0.5$ ) overlap because the bubble deforms slowly, and the speed of the surrounding liquid (although the internal gas velocity is quite large) and the generated magnetic resistance force in the surrounding conducting liquid are small. Thus, the effect of the magnetic field is small and the viscous and surface tension effects dominate during this early deformation.

The bubble shape after the initial deformation and the associated rise velocity of the 7 mm bubble with  $B = 0$  T are shown in Fig. 14. It takes about 0.85 dimensionless time (equivalent to 0.023 s) for the 7 mm bubble to achieve 63% of the terminal rising velocity (considering terminal velocity  $w^* \approx 1.35$ ). The momentum response time  $\tau_m$  of a 7 mm spherical bubble in Stokes flow can be obtained using

$$\tau_m = \frac{\rho_g d^2}{18\mu_l}, \quad (14)$$

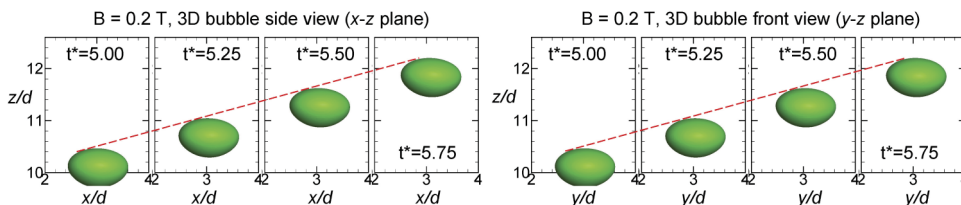
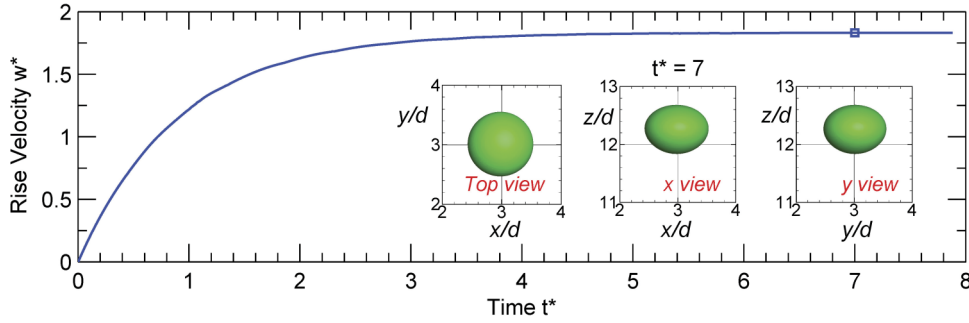


FIG. 11. Bubble shape at different times  $t^*$ ,  $d = 3$  mm,  $B = 0.2$  T.

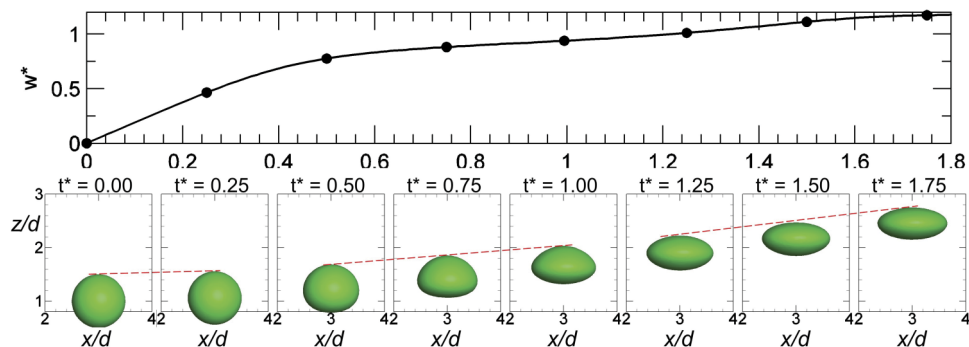
FIG. 12. Rise velocity and different views of the 3 mm bubble with  $B = 0.5$  T.

which gives  $\tau_m = 2.4 \times 10^{-4}$  s. This response time is almost 100 times smaller than the observed response time of  $2.3 \times 10^{-2}$  s. This result is also in agreement with a previous study<sup>18</sup> which observed that it takes 0.02–0.03 s for a large argon bubble ( $Eu = 2.2$ ) rising in GaInSn to reach 63% of the terminal rising velocity. The blue numbers in the sub-plots of Fig. 14 record the maximum number of cells in the cross section area of the bubble. The rise velocity is inversely related to cross-sectional area as indicated by those numbers.

### 3. Shape oscillations of the 7 mm bubble

For this bubble of larger diameter, the Eötvös number equals 2.80 larger than that of the 3 mm bubble ( $Eu = 0.51$ ). Therefore more deformations are seen when compared with those of the 3 mm bubble. As shown in Fig. 14, the 7 mm bubble first undergoes a symmetrical deformation with the bottom part of the bubble moving up and forming an ellipsoid at  $t^* = 2$ . Subsequently, the bubble deforms asymmetrically with the largest cross section diameter of  $1.7d$ . Consequently, the frontal area of the bubble has increased, exerting a larger drag on the bubble. The bubble rise velocity reaches a plateau at  $t^* = 2$ . After  $t^* = 2$ , the bubble deforms differently and the frontal area reduces at  $t^* = 3.25$ , giving a higher rise velocity. At  $t^* = 5.0$  the bubble is elongated along the diagonal of the duct with an increase in cross section. The bubble still remains at the center of the duct with a decrease in rise velocity. Further in time, at  $t^* = 8.24$  the bubble cross sectional area is reduced again with an elongated axis in the  $x$  direction. The elongation switches axes at  $t^* = 9.24$ , and the area of the cross section again increases.

As stated above, the oscillation of the rise velocity is linked to the bubble shape oscillation as well as the cross section area of the bubble. Fig. 14 shows that the bubble rise velocity varies with an approximate dimensionless cycle time period of  $t^* = 1.4$ . Using  $t = t^* g^{1/2} d^{1/2}$  we can convert this to a real time of 0.037 s, which corresponds to a frequency around 27 Hz. A previous experimental study<sup>39</sup> also found that when rising in purified water, the shape of a 5.5 mm air bubble oscillates with a frequency between 20 and 30 Hz. Assuming the waves to be capillary waves traveling on

FIG. 13. Bubble shape at different time  $t^*$ ,  $d = 7$  mm, without magnetic field.

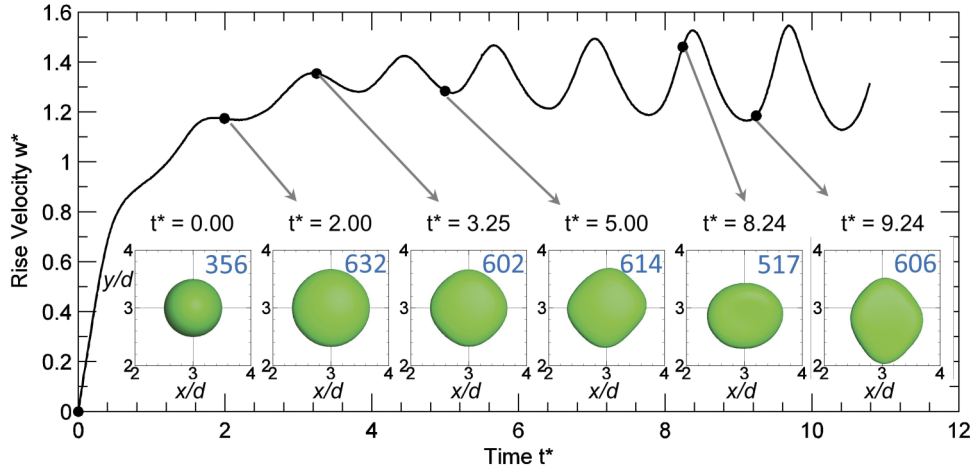


FIG. 14. Bubble rise velocity and the associated top-views of the 7 mm bubble at different times (shown as black dots on the line plot)  $B = 0$  T.

the bubble surface, Lunde and Perkins<sup>40</sup> obtained an expression for the mode (2,0) shape oscillation frequency,

$$f_{2,0} = \frac{1}{2\pi} \left( \frac{16\sqrt{2}\chi^2}{(\chi^2 + 1)^{3/2}} \right)^{1/2} \left( \frac{\gamma}{\rho_l(0.5d)^3} \right)^{1/2}, \quad (15)$$

where  $\chi$  is the ratio between the major and minor axes. In the present simulation without the magnetic field,  $\chi = 2.1$  is obtained from the side view of the 7 mm bubble. Substituting this in Eq. (15) gives  $f_{2,0} = 28.3$  which agrees very well with the present shape oscillation frequency of 27 Hz. However,  $\chi$  varies a lot during the rise of 7 mm bubble, and therefore it is valuable to obtain the range of the  $f_{2,0}$ . Figure 15 shows that  $\chi$  increases between 1 and 3,  $f_{2,0}$  varies in between (25.6 and 29.7). Therefore, in the argon-steel system, Eq. (15) is not very sensitive to the value of  $\chi$ , and the oscillation frequency is dominated by  $\gamma^{1/2}\rho_l^{-1/2}(0.5d)^{-3/2}$  term. Without the magnetic field, a maximum  $\chi = 2.63$  is found for the 7 mm bubble.

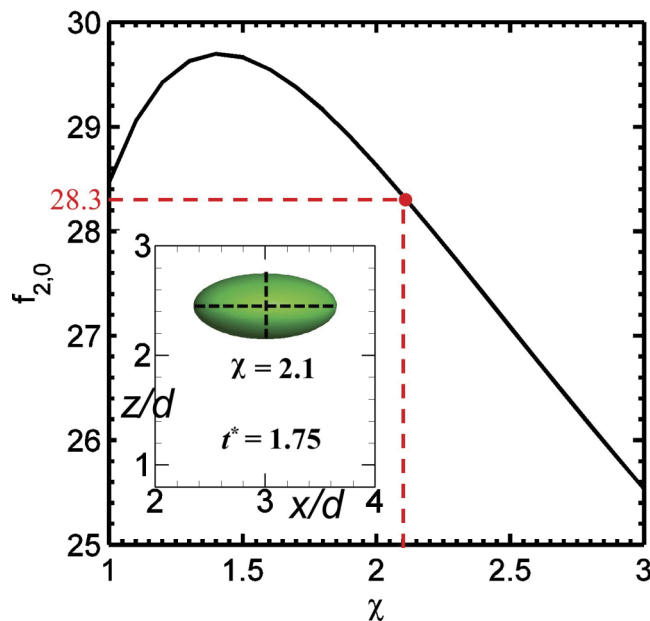


FIG. 15. Range of the shape oscillation frequency  $f_{2,0}$ .

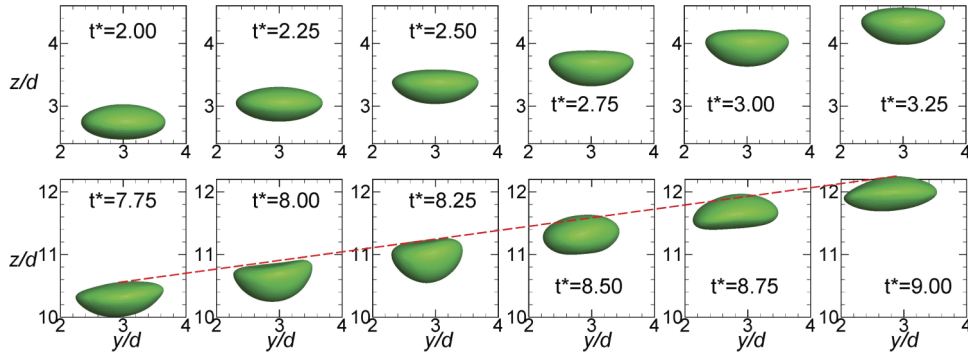
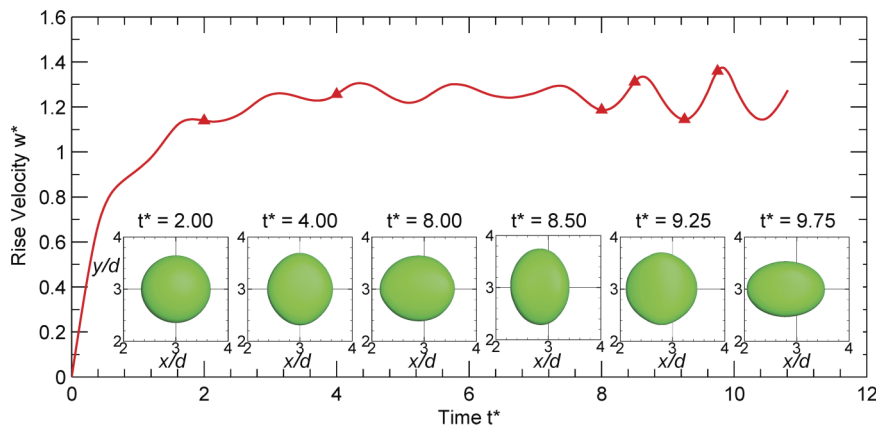
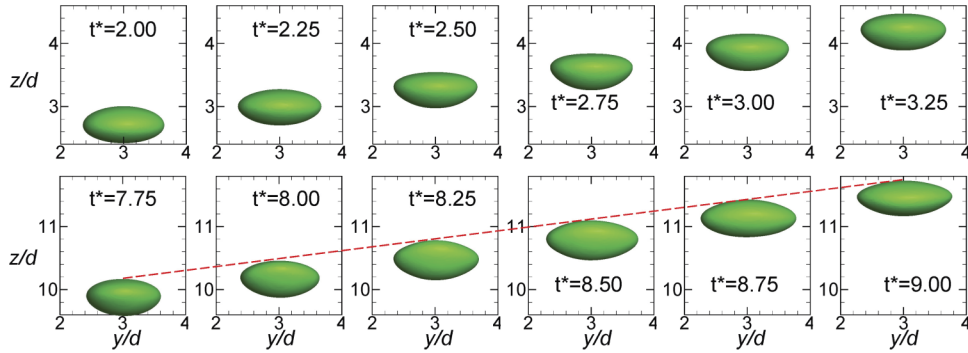
FIG. 16. Front views of the 7 mm bubble at different times  $t^*$ , without magnetic field.

Figure 16 shows the side views of the 7 mm bubble rising without the magnetic field. From  $t^* = 2$  to 3.25, the curvature at the front of the bubble becomes smaller while the curvature of the back becomes larger. The bubble thickness is slightly increased with time and the bubble remains in the center of the duct. The side views of the bubble from  $t^* = 7.75$  to 9 indicate that the bubble deforms considerably, reflecting in the rise velocity history plot shown in Fig. 14. We can see that a larger thickness of the bubble is usually associated with a larger rise velocity. The graphical method is applied again to estimate the average rising velocity of the bubble during this time period. It is seen that the dashed line is basically tangent to the surface of the bubble between  $t^* = 7.75$  and 8.25 but it passes inside of the bubble for  $t^* = 8.50$  and 8.75. This indicates that a slightly smaller average rise velocity should be obtained during the time period from  $t^* = 8.5$  to 9, which again agrees with the rise velocity curve shown in Fig. 14. These figures also show that the bubble is biased away from the centerline of the duct with the front of the bubble tilted and the bubble moving toward one side of the duct. The bubble also wobbles considerably during its rise. Although a larger deformation is seen for the 7 mm bubble, the bubble does not deviate from the centerline too much and the front face of the bubble does not tilt as much as that of the 3 mm bubble.

#### 4. Effect of magnetic field on shape of the 7 mm bubble

As shown in Fig. 17, when a 0.2 T magnetic field is applied, the rise velocity of the bubble is initially reduced by a small amount and fluctuates around  $W^* = 1.2$ . As in the case without the magnetic field, first a symmetrical deformation of the bubble occurs and the bubble shape changes from a sphere to spheroidal disk shape (squeezed along  $z$  direction) at  $t^* = 2$  and a decrease in rise velocity is seen. At  $t^* = 4$  the bubble is seen to be elongated in the  $y$  direction while remaining at the center of the duct. Then at  $t^* = 8$  the bubble is seen to be elongated in the  $x$  direction and

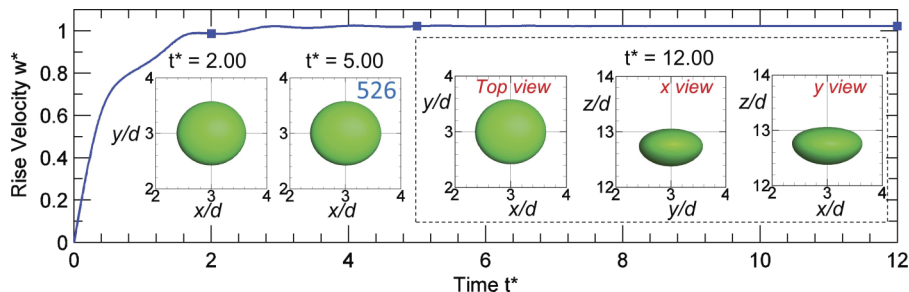
FIG. 17. Bubble rise velocity and the associated top-view of the 7 mm bubble at different times (shown as a red triangle on the line plot) with  $B = 0.2$  T.

FIG. 18. Front views of the 7 mm bubble at different times  $t^*$ ,  $B = 0.2$  T.

the  $x$  dimension of the bubble is  $\sim 1.4d$ , while the  $y$  dimension is seen to be  $1.2d$ . At  $t^* = 8.5$ , the direction of elongation switches to the  $y$  direction with the  $y$ -dimension becoming  $1.5d$  and the  $x$  dimension reducing to  $1.1d$ . The rise velocity reaches its local peak value of 1.35. The oscillation of the bubble shape continues with the major and minor axes switching directions, and the drag as well as rise velocity changing accordingly. The rise velocity at  $t^* = 9.75$  is close to 1.39. The local wake interacts with the magnetic field, thus linking several nonlinear phenomena together.

Front views of the 7 mm bubble (in the  $y$ - $z$  plane) under the magnetic field of 0.2 T are shown in Fig. 18. These views show that the bubble rises mainly vertically and not much motion in the  $y$  direction is observed. In the early stage ( $t^* < 3.25$ ), the shape of the bubble is seen to be a squeezed ellipsoid but later a more complex and time dependent bubble shape is developed. This alternative elongation behavior seen in Fig. 17 can also be found through analysis of the side views in Fig. 18. Comparing the front views at  $t^* = 7.75$  and  $t^* = 9$ , the thickness of the bubble did not change much but the dimension of the bubble along  $y$  is larger at  $t^* = 9$ . Since the volume of the gas is conserved, the bubble is relatively longer in the  $x$  direction at  $t^* = 7.75$ . These front views also show that the bubble is not tilted. Comparing the  $z$  locations of the bubble we see that the bubble rise velocity is slightly smaller than the value with no magnetic field. The graphical method shows an average rise velocity of 1.2 during the time period between  $t^* = 7.75$  and 9. This value is close to the average bubble rise velocity shown in Fig. 17.

Fig. 19 shows results for a further increase of the magnetic field to 0.5 T. We now observe a stable rise of the bubble and no time-dependent oscillations in rise velocity are seen. The bubble is slightly elongated in the direction of the magnetic field ( $x$  direction). The bubble size in the  $x$  direction is  $1.24d$ , and  $1.16d$  in the  $y$  direction. The shapes in the  $x$ - $z$  and  $y$ - $z$  planes are shown in the inset at  $t^* = 12$  where it is seen that the bubble is well centered in the duct. The bubble oscillations seen earlier are suppressed and the steady rise velocity is reduced to 75% of that without a magnetic field. Comparing these side views with the side views of the 7 mm bubble under  $B = 0$  and 0.2 T shown previously in Fig. 18, it can be observed that the bubble is thicker when the 0.5 T transverse magnetic field is applied.

FIG. 19. Bubble rise velocity and the associated top-view of the 7 mm bubble at different time (shown as the blue square on the line plot) with  $B = 0.5$  T.

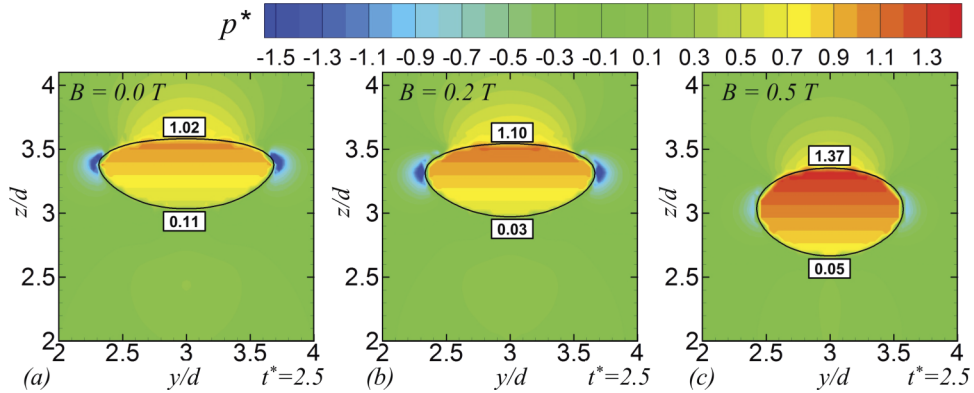


FIG. 20. Contours of  $p^*$  at  $t^* = 2.5$ ,  $d = 7$  mm, at middle plane  $x/d = 3$ : (a)  $B = 0$ ; (b)  $B = 0.2$  T; (c)  $B = 0.5$  T.

The maximum number of cells across the bubble is 526 which is smaller than that when no magnetic field is applied (Fig. 14), however the rise velocity is 25% smaller than that without magnetic field. The reason for this is that the Lorentz force resists the motion of the liquid steel and makes it appear to be more viscous. Thus, more energy is required to move the surrounding liquid steel, causing a lower bubble rise velocity.

## B. Pressure and velocity fields and Lorentz force distribution

In Subsections IV A 1–IV A 4, the bubble rise velocities and the deformations of the bubble were presented. Due to the close coupling between the velocity field and the induced Lorentz force, in this subsection we first show the pressure and velocity fields inside and adjacent to the bubble, and present the distribution of the Lorentz force, then demonstrate how the velocity field and bubble shape are affected by the Lorentz force.

The pressure field in the vicinity of the bubble is shown in Fig. 20 for the three cases of the 7 mm bubble. The pressures at the top and bottom of the rising bubble are shown in text. It can be seen that the maximum pressure difference increases with increasing  $N$ , in good agreement with a previous result.<sup>18</sup> This larger pressure drop across the bubble increases the drag acting on the bubble, and thus reduces its rising speed. Low pressure regions exist on the sides of the bubble, where maximum curvature is found. These pressures have an arbitrary level, hence only the difference is important.

The Lorentz force is generated due to the motion of the surrounding liquid (see Eq. (2)). Since the bubble is insulated, no current can pass through the bubble and therefore the damping effect of the Lorentz force can only affect the bubble by modifying the velocity of the surrounding conducting liquid. The velocity field of the gas phase and the surrounding liquid phase under different magnetic field values for the 7 mm bubble are shown in Fig. 21 at  $t^* = 2.5$ . In all three cases, a recirculation pattern can be seen at the boundary of the bubble with streamlines pointing outwards at the top half of the bubble and pointing inwards at the bottom half of the bubble. Comparing the

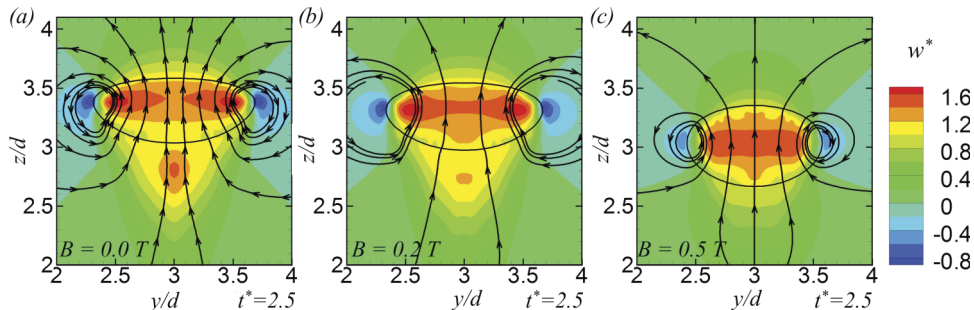


FIG. 21. Contours of  $w^*$  at  $t^* = 2.5$ ,  $d = 7$  mm, middle plane  $x/d = 3$ : (a)  $B = 0$ ; (b)  $B = 0.2$  T; (c)  $B = 0.5$  T.

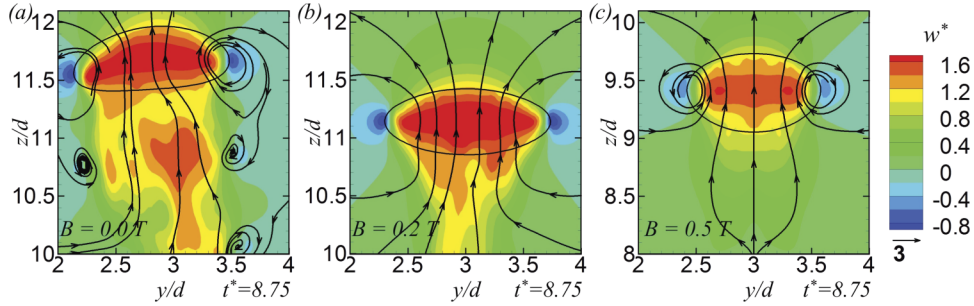


FIG. 22. Contours of  $w^*$  at  $t^* = 8.75$ ,  $d = 7$  mm, middle plane  $x/d = 3$ : (a)  $B = 0$ ; (b)  $B = 0.2$  T; (c)  $B = 0.5$  T.

cases of  $B = 0$  and  $B = 0.2$  T, when the magnetic field is applied, the maximum vertical velocity  $w^*$  at the bottom of the bubble is reduced from 1.6 to 1.3. With further increase of the magnetic field to 0.5 T, the bubble becomes thicker and less squeezed in vertical direction. The maximum value of  $w^*$  of the liquid outside the bubble is reduced from  $-0.8$  to  $-0.4$ . The velocities inside the bubble do not generate much Lorentz force because they are nearly insulated, therefore not much velocity reduction is seen inside of the bubble.

The contours of  $z$  direction velocity ( $w^*$ ) and path lines in the  $y$ - $z$  plane at  $t^* = 8.75$  for different magnetic fields are presented in Fig. 22. The recirculation is still present for all three values of  $B$ , but the shapes of the bubble and the rise velocities behind the bubble are different. When no magnetic field is applied, the bubble is no longer symmetrical and rises as a wobbling disk with unsymmetrical vortices behind. The  $w^*$  velocity behind the bubble is also unsymmetrical and it shows that  $w^*$  velocity at the right side ( $y > 3$ ) is higher than the left side. The bubble moves biased to the left side (towards the  $y$  direction). Considering the upcoming liquid at the bottom of the bubble as an impinging flow on the bubble, the flow below the right half of the bubble has a higher velocity. When the flow impinges on the bottom of the bubble, the bubble is pushed upward. Since the right side has a larger velocity, a larger pressure on the bubble is exerted and the bubble is biased to the left side. However, when the magnetic field is applied, there are no vortices shed and the flow velocity in the liquid is reduced with less asymmetry in the path of the bubble. Thus, the rise path is more rectilinear.

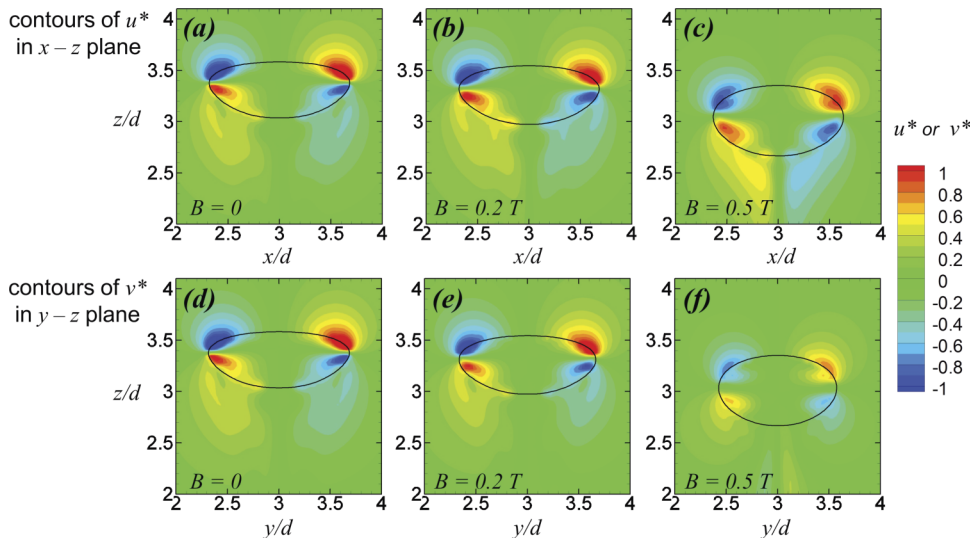


FIG. 23. Contours of  $u^*$  in the central  $x$ - $z$  plane (top) and contours of  $v^*$  in the central  $y$ - $z$  plane (bottom) for the three different magnetic field strengths at  $t^* = 2.5$ .

The contours of  $u^*$  velocity in the central  $x$ - $z$  plane and  $v^*$  velocity in the central  $y$ - $z$  plane for the three different magnetic field values at time  $t^* = 2.5$  are presented in Fig. 23. Figs. 23(a) and 23(d) show that when no magnetic field is applied, the distribution of  $u^*$  velocity in the central  $x$ - $z$  plane is similar to the distribution of  $v^*$  velocity in the central  $y$ - $z$  plane, implying symmetry. When a magnetic field of  $B = 0.2$  T is applied, Figs. 23(b) and 23(e) show still nearly similar distributions as those without the magnetic field. However, with further increase of  $B$  to 0.5 T, a different velocity distribution is seen in Figs. 23(c) and 23(f). Comparing the two figures, the high  $u^*$  velocity region in  $x$ - $z$  plane is larger than the high  $v^*$  velocity region in the  $y$ - $z$  plane, with larger maximum  $u^*$  than maximum  $v^*$ . The horizontal motion of the fluid in the  $x$  direction is more intensive than that in the  $y$  direction. In the momentum equations (Eq. (2)), although the induced Lorentz force  $\mathbf{F}_L$  consists of three components ( $\mathbf{F}_{Lx}, \mathbf{F}_{Ly}, \mathbf{F}_{Lz}$ ), the  $x$  component of the force  $F_{Lx}$  equals zero since it is parallel with the magnetic field direction. Therefore, the Lorentz force cannot directly affect the motion in the  $x$  direction. However, the damping effect of the Lorentz force in the  $y$  and  $z$  directions affects the velocities in those directions and modifies the  $x$  direction velocity by continuity condition. The distribution of  $v^*$  velocity in the central  $y$ - $z$  plane is different from the other cases with  $B = 0$  or 0.2 T, and the contours of  $v^*$  show two tails following the bubble. Comparing Figs. 23(a)–23(c) we can see that the  $u^*$  velocity behind the bubble is significantly increased at  $B = 0.5$  T and it is larger than the cases of  $B = 0$  and 0.2 T. The Lorentz force does not directly affect the flow inside the bubble due to the almost zero conductivity of the gas, but under the strong magnetic field of 0.5 T Figs. 23(c) and 23(f) show that the circulation inside the bubble in the  $y$ - $z$  plane is reduced compared to the circulation in the  $x$ - $z$  plane. This is caused by the suppression of  $y$  direction motion of the surrounding fluid. Therefore, the magnetic field can also reduce the circulation inside of the bubble indirectly by affecting the surrounding liquid.

Since the Lorentz force modifies the velocity distribution, in Fig. 24 we show the distributions of Lorentz force in the  $y$ - $z$  plane ( $x/d = 3$ ) for the two different  $B$  values at the same dimensionless time  $t^* = 2.5$ . The first plot in Fig. 24 showing contours of the  $y$  component of the Lorentz force together with lines shows the direction of the Lorentz force. It is seen that on the top half of the bubble the force is pointing towards the inside of the bubble and tries to squeeze the bubble along  $y$  and  $z$  directions. However, at the bottom half of the bubble the  $y$  component of the Lorentz force is positive on the right side but negative on the left side, which means that the force is trying to pull the liquid away from the bubble. The contour plots of  $z$  component of the Lorentz force shows that the Lorentz force is pushing the liquid at the top of the bubble downward and also pulling liquid downward at the bottom of the bubble. The force also decelerates the steel flow at the side of the bubble, thus diminishing the recirculation. All these effects can be more clearly seen when  $B$  is 0.5 T. The contours of  $F_y^*$  also show that the force tends to push the liquid inward in the upper portion of the bubble and pull the liquid away from the bubble in the bottom portion. The two tails in the contours of  $F_y^*$  are caused by the direction of the  $y$  component of the velocity behind the bubble. The contour plot of  $F_z^*$  at  $B = 0.5$  T shows that the Lorentz force resists the rise of the bubble. In the  $y$ - $z$  plane the maximum  $F_z^*$  at the bottom of the bubble is not at the center but slightly biased to one side.

Consistent with earlier studies<sup>18,19,21–23</sup> we have also seen that the bubble is elongated along the magnetic field direction. However, the mechanism by which the bubble is elongated has not been

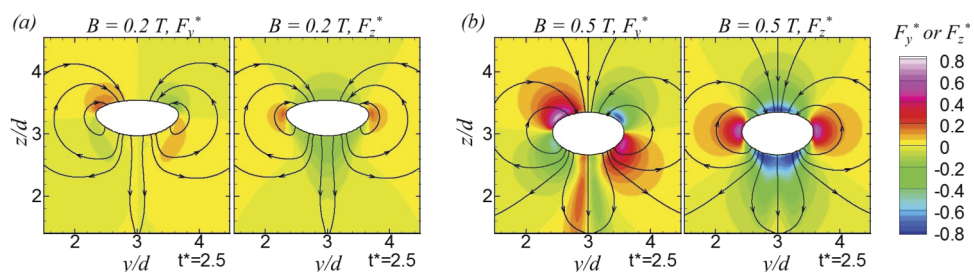


FIG. 24. Contour plots of  $F_y^*$  and  $F_z^*$  at  $t^* = 2.5$  and direction of Lorentz force, for bubble  $d = 7$  mm at middle plane  $x/d = 3$ : (a)  $B = 0.2$  T, (b)  $B = 0.5$  T.

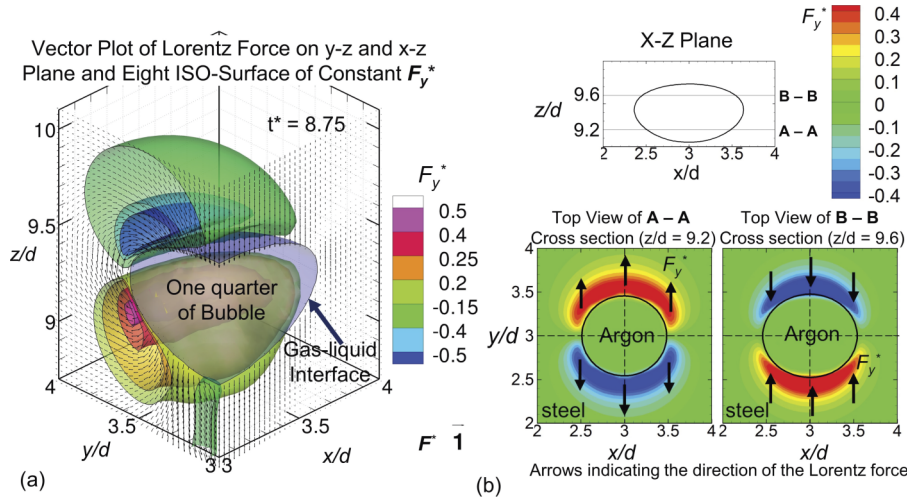


FIG. 25. (a) Vectors of Lorentz force in the  $y$ - $z$  plane and  $x$ - $z$  plane and eight isosurfaces of constant  $F_y^*$ . (b) Contours of  $F_y^*$  at the top and bottom half of the rising 7 mm bubble,  $B = 0.5$  T and  $t^* = 8.75$ .

previously fully understood. Figure 25 shows the isosurfaces of force in  $y$  direction  $F_y^*$  and contours in two different planes. Analysis of these plots demonstrates that the bubble's elongation should be rather expressed as “bubble compression” in the perpendicular plane, and the compression is caused by the anisotropic distribution of the Lorentz force. The left plot in Fig. 25 shows one quarter of the bubble (represented by a blue surface at the center) and the isosurfaces of constant  $F_y^*$ . In addition, the Lorentz force vectors in the quarter planes ( $y$ - $z$  plane and  $x$ - $z$  plane) are shown. The isosurface plot shows that  $F_y^*$  reaches its maximum value in the  $y$ - $z$  plane and decays as we rotate it towards becoming the  $x$ - $z$  plane. This is because the velocity component in  $y$  direction is decreasing, and in  $x$ - $z$  plane the  $F_y^*$  is zero due to it being nearly symmetrical. Therefore only the  $F_z^*$  component exists in the  $x$ - $z$  plane. Hence the force vectors in the  $x$ - $z$  plane only point upward or downward depending on the direction of the  $z$  velocity. However, due to the existence of  $F_y^*$  in the  $y$ - $z$  plane the vectors are in  $y$  and  $z$  directions. As the bubble rises in the duct, the top of the bubble continually pushes away the surrounding liquid and if no transverse magnetic field is applied, an axisymmetric shape of the bubble is expected. In that case the liquid should be pushed away uniformly in the radial direction at the top half of the bubble and pulled inward at bottom half of the bubble. Thus, there should be no difference between  $x$  and  $y$  directions of the bubble shape, unless instabilities trigger and lead to an asymmetrical shape. After applying the magnetic field in the  $x$  direction, a Lorentz force component  $F_y^*$  is generated in the domain which acts as an additional resistance force along  $y$  direction and tries to prevent the liquid being pushed away along the  $y$  direction. This leads to an additional compressive force that acts on the bubble. The bubble cannot push the surrounding liquid away along the  $y$  direction as easily as it can push the liquid along the  $x$  direction. Therefore the bubble ends up shorter in the  $y$  direction compared to the  $x$  (magnetic field) direction. It is therefore appropriate to interpret that the bubble “elongation in the magnetic field direction” is actually caused by a compression in the direction perpendicular to the field.

### C. Flow structures behind the bubble

The non-rectilinear motion of the bubble has been previously related to the structure of the wake formed behind the bubble.<sup>18,38,41</sup> It was proposed that these instabilities in the wake cause an asymmetrical flow behind the bubble and further lead to a zig-zag or helical motion of the bubble. In this section, we present the observed wake structures behind the bubble and discuss the changes caused by the magnetic field on these structures. Fig. 26 shows the front and side views of the vorticity magnitude of a value  $|\omega^*| = 3$  at a representative time  $t^* = 5.75$  for the three different magnetic field values. These plots show that the wake structure behind the bubble has a “tail-like” shape. A comparison of the lengths of the tails shows that with increase of the magnetic field strength, the

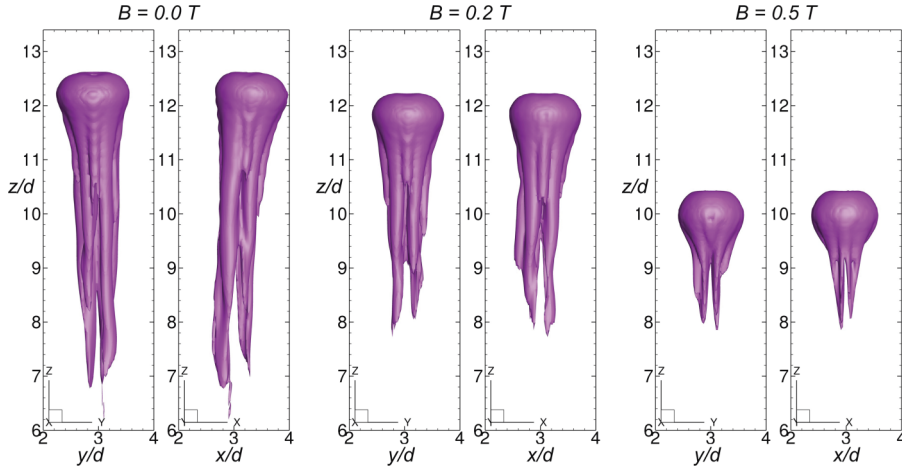


FIG. 26. Front and side views of the isosurfaces of  $|\omega^*| = 3$  at  $t^* = 5.75$  under different magnetic field strengths, original bubble diameter  $d = 3$  mm.

wake becomes smaller. The reduction of the wake region is caused by two effects: (1) the external magnetic field induces a Lorentz force which increases the drag experienced by the bubble, causing a reduction in rise velocity and consequently slower motion of the surrounding liquid and (2) the flow motion in the wake region is suppressed by the induced Lorentz force.

Isosurfaces of  $\omega_z^* = \pm 1$  at  $t^* = 5.75$  for the 3 mm bubble are shown in Fig. 27. These figures show that  $\omega_z^*$  surrounding the bubble alternates in sign, which indicates that the rotation of the fluid in the  $z$  direction consists of alternating rotating pairs. In the case without the magnetic field, the side views ( $y$ - $z$  plane) show that the bubble is slightly biased away from the center and rises towards the  $y$ - side of the duct. The  $z$  vorticity is seen to be unsymmetrical and the tail on the  $y$ + side is slightly longer. The front view ( $x$ - $z$  plane) shows the bubble is biased from the center and rises towards the  $x$ + side. Again, the  $z$  vorticity is seen to have a longer tail on the opposite side ( $x$ - side). After applying the magnetic field, the bias of  $z$  vorticity disappears and the bubble is seen to rise rectilinearly. Surprisingly, upon further increase of the magnetic field to 0.5 T the affected region of the  $z$  vorticity is seen to become larger, indicated by a longer tail following the bubble. Figure 27 also shows that with  $B = 0.5$  T the  $z$  vorticity behind the bubble does not alternate as many times as seen for  $B = 0$  and 0.2 T. However, the top views of the  $z$  vorticity in Fig. 28 show that in all these cases there are 4 rotation pairs in the front and surrounding the bubble.

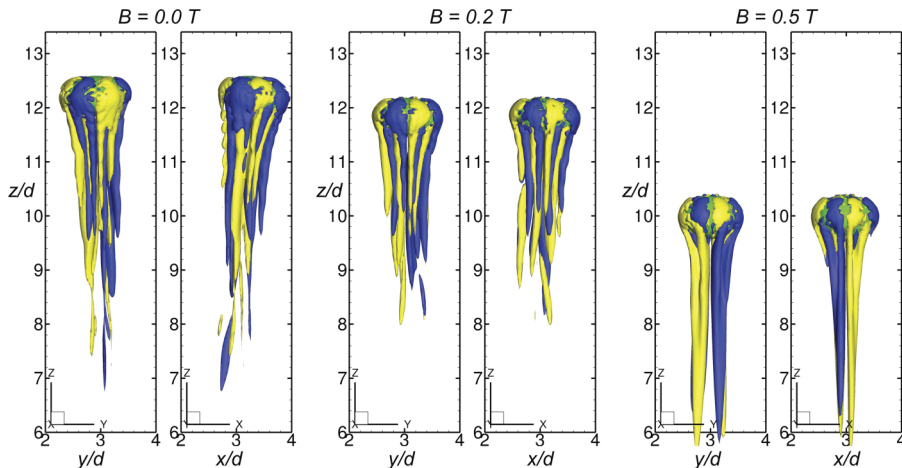


FIG. 27. Front and side views of the bubble (green), isosurfaces of  $\omega_z^* = 1$  (bright yellow), and  $\omega_z^* = -1$  (dark blue) at  $t^* = 5.75$  for 3 mm bubble.

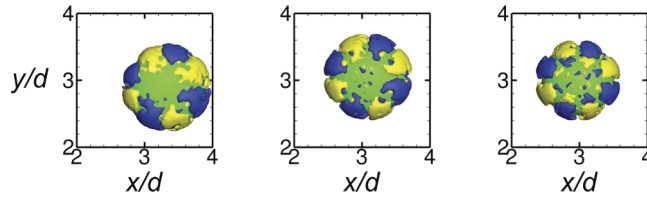


FIG. 28. Top views of the bubble (green), isosurfaces of  $\omega_z^* = 1$  (bright yellow) and  $\omega_z^* = -1$  (dark blue) at  $t^* = 5.75$  of 3 mm bubble for  $B = 0$  T,  $B = 0.2$  T and  $B = 0.5$  T (from left to right).

The isosurfaces of  $|\omega^*| = 4$  at  $t^* = 9$  for  $d = 7$  mm and different magnetic fields are shown in Fig. 29 (see Multimedia view options in the caption for Fig. 29 to view animations of these isosurfaces). With the larger bubble diameter, the wake structures behind the bubble are more complex and hairpin structures are seen.<sup>15,23,42</sup> When no magnetic field is present, the isosurface shows interconnected hairpin structures behind the bubble and they persist even after  $5d$  behind the rising bubble. These structures indicate that a larger shear is experienced by the bubble. These hairpin structures are probably generated by the shape oscillations of the bubble, as no such structures are seen for the 3 mm bubble or in the initial rise of the 7 mm bubble. These hairpin structures are generated at similar frequency as the shape oscillation frequency  $f_{2,0}$ . The oscillatory bubble shape and complex wake structures behind the bubble lead to unsteady motion of the bubble. After a strong magnetic field is applied, the bubble shape is stable and the hairpin structures in the wake are suppressed. After applying a magnetic field of 0.2 T along the  $x$  direction, the vortex behind the bubble is compressed and the region affected in the  $x$  direction is slightly larger than that along the  $y$  direction. Although some hairpin-like structures are seen, they are smaller and more elongated in the magnetic field direction ( $x$  direction). The alternate shedding of these hairpin-like vorticities is related to the zigzag motion of rising bubbles.<sup>42</sup> With further increases of magnetic field strength to 0.5 T ( $N = 0.68$ ), the complex wake structures behind the bubble disappear due to the flow getting damped by the Lorentz force, and the bubble rises straight upward.

Figure 30 shows the isosurfaces of  $\omega_z^* = \pm 1$  at  $t^* = 9$  for the 7 mm bubble (see Multimedia view options in the caption for Fig. 30 to view animations of these isosurfaces). A pattern of alternating  $\omega_z^*$  is seen again but the structure behind is more complex than that seen behind the 3 mm bubble. With a magnetic field of 0.2 T, it is seen that the isosurfaces are elongated in the magnetic field direction ( $x$  direction). This effect is caused by the fact that by applying the magnetic field along the  $x$  direction the  $y$  and  $z$  velocities in the surrounding fluid are reduced, thus damping the flow perpendicular to the magnetic field direction. The damping effect of the external magnetic field of

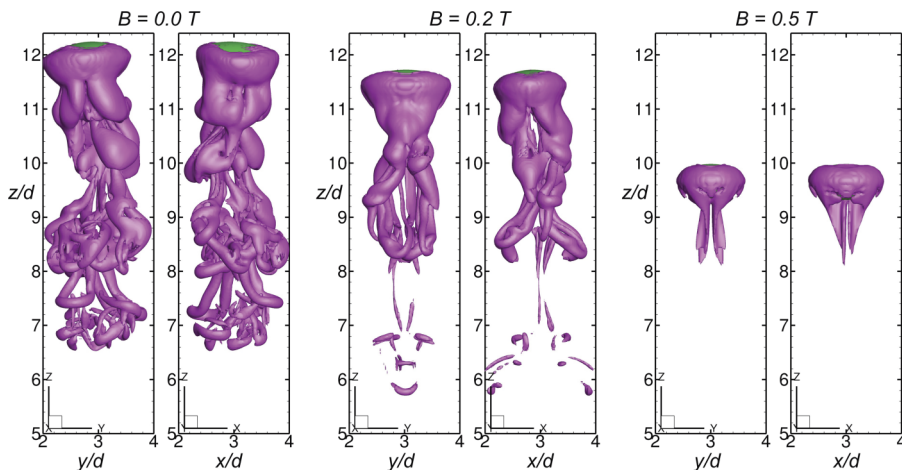


FIG. 29. Front and side views of the bubble (green), isosurfaces of  $|\omega^*| = 4$  (pink color) at  $t^* = 9$  for 7 mm bubble with  $B = 0$  T,  $B = 0.2$  T, and  $B = 0.5$  T. (Multimedia view) [URL: <http://dx.doi.org/10.1063/1.4961561.1>] [URL: <http://dx.doi.org/10.1063/1.4961561.2>]

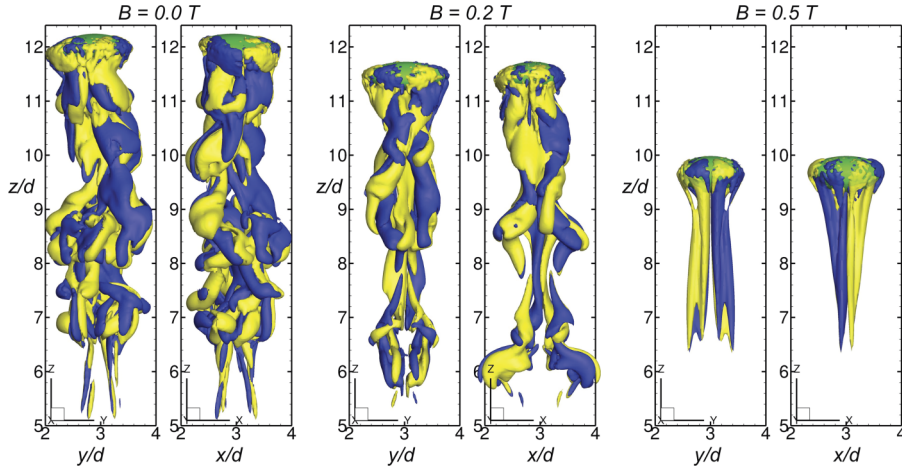


FIG. 30. Front and side views of the bubble (green), isosurfaces of  $\omega_z^* = 1$  (bright yellow) and  $\omega_z^* = -1$  (dark blue) at  $t^* = 9$  for 7 mm bubble with  $B = 0$  T,  $B = 0.2$  T, and  $B = 0.5$  T. (Multimedia view) [URL: <http://dx.doi.org/10.1063/1.4961561.3>] [URL: <http://dx.doi.org/10.1063/1.4961561.4>]

different strengths has been earlier studied in detail for flow in a driven cavity.<sup>32</sup> With increase of the magnetic field strength to 0.5 T, the complex wake structure is seen to nearly disappear. It is seen that the front view of the isosurface is wider (along  $x$  direction) in the region close to the bubble, while in the downward region the isosurfaces are bundled. However, the wake along  $y$  direction is not compressed but slightly spreads out.

#### D. Modified drag coefficient

In the steel casting industry, argon bubbles are often injected during the continuous casting process to prevent clogging and to remove inclusions (i.e., aluminum oxide).<sup>2,43</sup> A transverse static magnetic field up to 0.3 T is often used to optimize the flow pattern in the mold region.<sup>44,45</sup> Several numerical simulations<sup>43,45–47</sup> have been previously carried out to understand the transport of these argon bubbles in the caster. However, most models do not include the effect of Lorentz force on the drag coefficient. Since the present work shows that ignoring this effect may cause errors in the bubble velocities, a modified drag coefficient is calculated using present simulations.

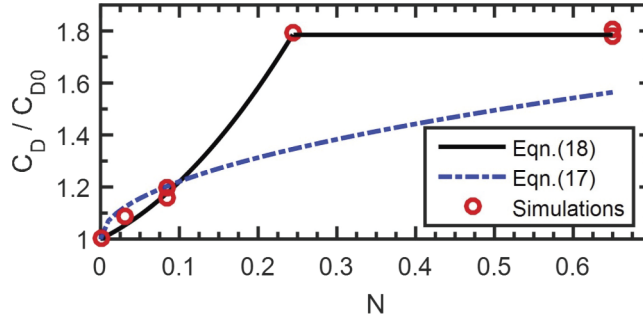
When the steady state is reached, the drag force acting on the bubble is balanced with the buoyancy force. For a fixed bubble volume, the buoyancy force is independent of  $N$ . Therefore the total drag force exerted on the bubble is a constant. Hence, one can write

$$C_D w^2 = C_{D0} w_0^2 = \text{Constant}, \quad (16)$$

where  $C_D$  is the drag coefficient and  $w$  is the bubble rise velocity, subscript “0” denotes the value obtained when no magnetic field is applied ( $N = 0$ ). The rise velocities of the bubble are known from the simulations, and then Eq. (16) is used to compute the estimated new  $C_D/C_{D0}$ . The results are presented in Table VII. Comparing the results for 5 and 7 mm bubbles with the same Stuart

TABLE VII. Effect of Stuart number  $N$  ( $Ha^2/Re_\tau$ ) on drag coefficient  $C_D$ .

| 3 mm bubble |             |              | 5 mm bubble |             |              | 7 mm bubble |             |              |
|-------------|-------------|--------------|-------------|-------------|--------------|-------------|-------------|--------------|
| $N$         | $w^*/w_0^*$ | $C_D/C_{D0}$ | $N$         | $w^*/w_0^*$ | $C_D/C_{D0}$ | $N$         | $w^*/w_0^*$ | $C_D/C_{D0}$ |
| 0           | 1.000       | 1.000        | 0           | 1.000       | 1.000        | 0           | 1.000       | 1.000        |
| 0.030       | 0.959       | 1.087        | 0.085       | 0.930       | 1.156        | 0.085       | 0.914       | 1.196        |
| 0.244       | 0.747       | 1.792        | 0.649       | 0.744       | 1.806        | 0.649       | 0.750       | 1.778        |

FIG. 31. Effect of  $N$  on the drag coefficient.

number, the predicted  $C_D/C_{D0}$  are slightly different, especially when  $N$  is small. This difference may be caused by the bubble shape and its oscillation. With  $N = 0.085$ , the 5 mm bubble rises without shape oscillations, but the shape of the 7 mm bubble still oscillates and terminates with an oscillatory rise velocity. With  $N = 0.649$ , both 5 and 7 mm bubbles rise rectilinearly and the calculated  $C_D/C_{D0}$  for both cases are very close.

Previous experimental work<sup>48</sup> of conducting fluid flow past rigid spheres suggests that

$$C_D = C_{D0} (1 + 0.7\sqrt{N}) \quad (17)$$

is suitable for the range  $17.6 < Re_\tau < 332$  and  $N < 2.5 \times 10^3$ . The present study shows the shape of the larger argon bubble is not spherical, and the  $Re_\tau$  number is higher than 1000, therefore Eq. (17) cannot be used directly. To obtain a modified drag coefficient, a curve fitting of the values in Table VII yields

$$C_D = \begin{cases} C_{D0}(1.0 + 1.50N + 7.06N^2), & \text{if } 0 \leq N < 0.245, \\ 1.8C_{D0}, & \text{if } 0.245 \leq N < 0.65. \end{cases} \quad (18)$$

The above curves suggest that  $C_D/C_{D0}$  increases with  $N$  when  $N < 0.25$ , and it reaches a plateau of  $C_D/C_{D0} = 1.8$  for  $N > 0.25$ . The above relation includes the effect of bubble shape deformation and works for low Morton number ( $Mo = 1.3 \times 10^{-12}$ ) system and  $Eo < 2.80$ . Equations (17) and (18) are also shown in Fig. 31 together with the simulation results. However, Eq. (17) under-predicts the modified drag coefficient when the magnetic effect is strong. Possible reasons for this discrepancy are (1) the bubble is deformable and the shape is not spherical; (2) the bubble  $Re_\tau$  is higher than the upper bound of 332. It is important to note that the present study shows applying a transverse magnetic field increases the drag coefficient by as much as  $\sim 1.8$  times.

## V. SUMMARY

In this paper we have studied the three dimensional dynamics of an argon bubble rising in molten steel in the presence of a transverse magnetic field. A VOF interface tracking method with sharp surface force treatment is used to reduce spurious velocities near the bubble. The code was validated by simulating the rise of an air bubble in water and the rise of an argon bubble in GaInSn. The predicted rise velocities matched very well with other published results. Then six simulations of argon bubbles of two different sizes rising in molten steel were performed with the shapes of the bubble well resolved. The results show that an external magnetic field reduces the rise velocity, suppresses bubble shape oscillations, and leads to a more rectilinear path. For a 3 mm argon bubble rising under a magnetic field of 0.2 T, the terminal rise velocity is seen to slightly increase compared with that of the no magnetic field case. This is caused by the fact that the rising path of the bubble is more rectilinear. Further increase of the magnetic field reduces the rise velocity. For a 7 mm bubble, the results show that the bubble undergoes large deformations and shape oscillations. The oscillatory bubble shape further affects its rise velocity as well as the wake behind. During the oscillations, the bubble rises slower with a larger cross section area. Shape oscillations also generate

hairpin structures in the wake. Without the magnetic field, the shape of a 7 mm bubble oscillates at a frequency around 28 Hz. The simulations also show that with a large magnetic field strength, the unsteady shape deformations are suppressed and the hairpin type complex wake structures behind the bubble almost disappear. The results also show that although the gas is not conducting, by modifying the fluid velocity adjacent to the bubble, the applied magnetic field reduces the circulation inside the bubble. The bubble is seen to be compressed in the direction perpendicular to the applied magnetic field direction. The observed results are a direct result of the Lorentz force distribution which increases the resistance to flow motion in the plane perpendicular to the magnetic field. A correlation of  $C_D/C_{D0}$  with the Stuart number is presented.

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