Technical Notes and Correspondence

Online Recalibration of the State Estimators for a System With Moving Boundaries Using Sparse Discrete-in-Time Temperature Measurements

Bryan Petrus, Zhelin Chen, Joseph Bentsman, *Member, IEEE*, and Brian G. Thomas

***Abstract*—In this paper, the problem of estimation is considered for a class of processes involving solidifying materials. These processes have natural nonlinear infinite-dimensional representations, and measurements are only available at particular points in the caster, each corresponding to a single discrete-in-time boundary measurement in the Stefan problem partial differential equation (PDE) mathematical model. The results for two previous estimators are summarized. The first estimator is based on the Stefan problem, using continuous instead of discrete-in-time boundary measurements. The second estimator employs a process model that is more detailed than the Stefan problem, but with no output injection to reduce estimation error, other than model calibration. Both of these estimation frameworks are extended in the current paper to a more realistic sensing setting. First, an estimator is considered that uses the Stefan problem under some simplifying but practically justified assumptions on the unknowns in the process. The maximum principle for parabolic PDEs is employed to prove that online calibration using a single discrete-in-time temperature measurement can provide removal of the estimation error arising due to mismatch of a single unknown parameter in the model. Although unproven, this result is then shown in simulation to apply to the more detailed process model.**

***Index Terms*—Casting, mathematical modeling, solidification, steel, stefan problem, temperature measurement.**

I. Introduction

Processes involving solidification are wide-spread in manufacturing, but pose several significant challenges to traditional control theoretic methods, as they are fundamentally infinite-dimensional and nonlinear in nature. The simplest, but still accurate, model of such processes, commonly called the Stefan problem, splits the spatial domain into separate subdomains for the liquid and solid parts of the material. Within the subdomains, temperature follows the usual parabolic heat-diffusion partial differential equation (PDE). The boundary between the domains moves according to conservation of energy, written as the Stefan condition in terms of the temperature gradients on both sides of the boundary.

Moreover, specific solidification manufacturing methods pose problems that are generally not considered within the field of distributed parameter control systems. Consider the process of continuous casting, which as of 2013 was used to make more than 90% of the steel in the world [1]. An illustration of this process is shown in Fig. 1. Continuous casting, as opposed to traditional casting methods, keeps a constant flow of liquid metal into the machine. The metal in the caster, called the strand, cools and solidifies as it moves through the machine. Below the mold, heat is removed in the secondary cooling zones by water cooling sprays and direct contact with support rolls. At the exit of the caster, the fully solid metal is cut into separate pieces either to be processed further or shipped directly to a customer. Regulation of surface temperature of the steel strand is vital to guaranteeing product quality and operational safety. The curved strand of steel must be unbent, which causes tensile stress on the inside radius surface that results in transverse cracks if the steel is too brittle. A common practice for preventing surface cracks is to adjust the spray water flow rates to keep the strand surface temperature at the bender/straightener either above or below this low ductility region.

The basic estimation problem for this system, estimating the distributed temperature profile within the strand using only boundary measurements, is difficult enough given the nonlinear governing PDE. However, the actual measurements available are typically sparse. The support rolls and the machinery of the caster itself block access to the strand surface in much of the caster. The rest of the surface is usually being sprayed with water to cool the metal. The sprays themselves and the steam where they hit the strand interfere greatly with optical pyrometers. A typical caster may have only one or two pyrometers in the entire machine. The accuracy of estimation of local spray water heat removal is significant for control of the spray cooling region.

In Section II, a basic mathematical model of solidification is described, based on a continuous steel slab caster. The specific difficulties of the estimation problem are described. In Section III, a brief description is given of some previous work on the subject. In Section IV, a new result is described that uses the temperature measurement to recalibrate, through changing one of the parameters, the existing temperature estimator. Since the latter provides an input to the real-time spray cooling controller, the estimate accuracy improvement attained guarantees better temperature control performance. Section V presents conclusion and future work.

II. Mathematical Models

Before introducing the actual PDEs, a brief discussion is needed on scaling analysis. The discussion to follow is based on [2], where more detailed information may be found. Inside the strand of a caster, heat is transferred by two methods: diffusion and advection. The latter is

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B. Petrus was with the University of Illinois at Urbana-Champaign, Champaign, IL 61801 USA. He is now with, Nucor Steel, Decatur, AL 35673 USA (e-mail: bjpetrus@gmail.com).

Z. Chen and J. Bentsman are with the University of Illinois at Urbana-Champaign, Champaign, IL 61801 USA (e-mail: zchen61@illinois.edu; jbentsma@illinois.edu).

B. G. Thomas is with the Colorado School of Mines, Golden, CO 80401 USA (e-mail: bgthomas@illinois.edu).

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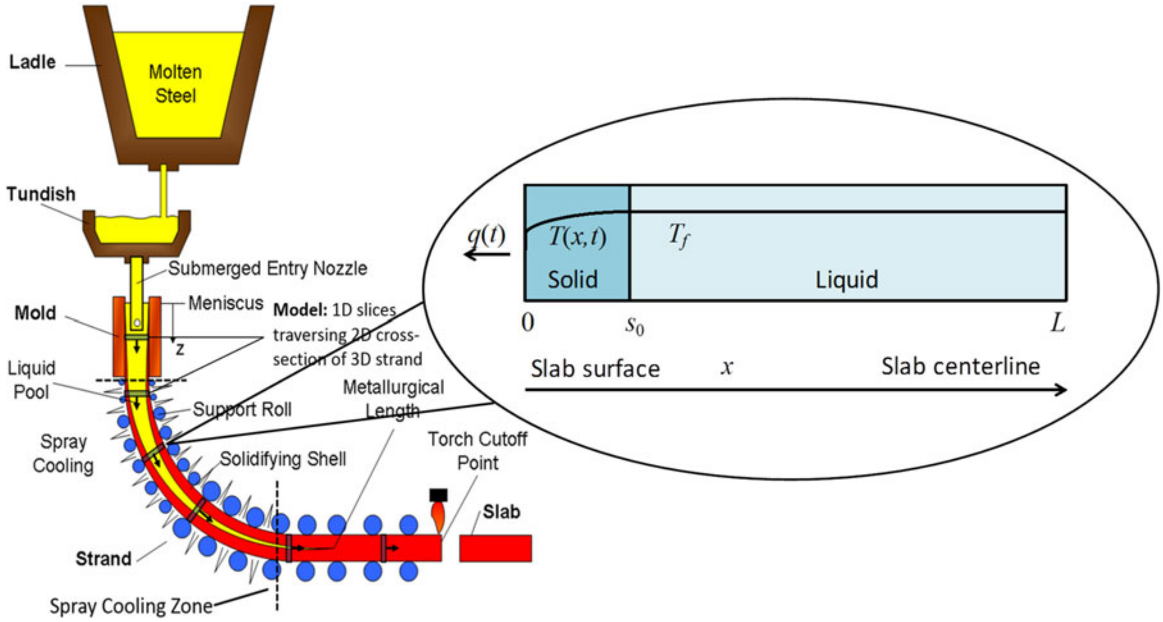

Fig. 1. Illustration of continuous steel slab caster.

heat transported by the actual movement of metal through the caster. At typical casting speeds, advection heat transfer is much faster than diffusion heat transfer, to the point where the latter is negligible in the casting direction. Rather than model the entire three-dimensional (3-D) domain, reasonable accuracy is achievable by modeling a two-dimensional (2-D) slice of the material as it moves down through the caster. Furthermore, in slab casters, i.e., when the aspect ratio of the 2-D slice is very large, heat transfer is dominant in the smaller transverse dimension. Therefore, a one-dimensional (1-D) slice gives good accuracy, and will be used for this work. For simplicity, the work in this paper will also assume the temperature is symmetric across the center of the strand. This will simplify the notation, and the results can mostly be generalized straightforwardly.

This 3-D-to-1-D dimension reduction is important to the present work because changing the frame of reference changes the nature of the measurement. In the full 3-D reference frame, a pyrometer is a point measurement in *space*. In the 1-D reference frame, a pyrometer is instead discrete in *time*, $\int_0^{z^*} dz/V_c$ taken when the slice passes beneath the location z^*. where the pyrometer is installed. The common control theoretic concept of estimation assumes measurement over a nonzero length of time, and so many existing results cannot be directly applied to this problem.

A. Stefan Problem

The Stefan problem [3] models a solidifying material, in this case the moving 1-D slice of the caster, by dividing it into two separate subdomains, solid and liquid. Within each subdomain, temperature evolves according to the usual linear parabolic heat diffusion equation. The boundary between the two domains moves according to conservation of energy between the heat fluxes—proportional to the temperature gradients—on either side of the boundary and the latent heat of solidification.

Denote x to be the spatial variable, t to be the time variable, $T(x,t)$ to be the temperature, and $s(t)$ to be the location of the liquid–solid interface. In the equations to follow, subscripts of x and t indicate partial derivatives. In general, arguments will not be included to simplify notation. Then, the Stefan problem is written as

$$T_t = aT_{xx}, \quad x \in (0,s) \cup (s,L), \tag{1}$$

$$T(s,t) = T_f, \tag{2}$$

$$s_t = \frac{k}{\rho L_f}\left(T_x\left(s^-(t),t\right) - T_x\left(s^+(t),t\right)\right) \tag{3}$$

where the material is solid for $x \in (0,s)$ and liquid for $x \in (s,L)$, L is the half-thickness of the slab, T_f is the melting temperature, a is the thermal diffusivity $k/\rho c_p$, and the properties k (thermal conductivity), ρ (density), c_p (specific heat), and L_f (latent heat) can vary with temperature (see [2]). For the specific 1-D slice problem, the initial conditions (ICs) and boundary conditions (BCs) are

$$T(x,0) = T_0, \; s(0) = s_0, \tag{4}$$

$$T_x(L,t) = 0, \tag{5}$$

$$T_x(0,t) = q(t). \tag{6}$$

The boundary heat flux q will be discussed below.

One assumption will be made to simplify the problem:

(A1) $T_0(x) \le T_f, \quad x \in (0,s_0)$ and $T_0(x) = T_f, \quad x \in (s_0,L)$.

That is, the material is initially below the melting temperature in the solid and equal to the melting temperature in the liquid. The first condition is physically necessary. The second condition is simplistic, but not overly so. The temperature superheat (the temperature increment above the melting temperature) entering the mold from the tundish is typically monitored online at the steel plant. Most of this superheat is distributed relatively uniformly in the liquid region according to the turbulent flow pattern in the mold, and removed by the time the molten steel reaches the meniscus at the top of the mold, where solidification of the solid shell begins. The superheat in the liquid in a caster is around 25 °C to 50 °C, while the temperature at the strand surface is hundreds of degrees below the melting temperature. Therefore, neglecting the temperature gradients in the liquid is a common simplification in modeling continuous casters. Since this limits the temperature transients to the solid area, this simplification is sometimes called the "single-phase"

Stefan problem. The slice domain corresponding to the latter is shown in the oval in Fig. 1.

In (4), the initial temperature distribution across $(0, L)$ is assumed known. The initial temperature of the liquid steel at the meniscus can be treated with very small error as uniform and equal to the pouring temperature. With this simplification, we actually have $s_0 = 0$. However, in the real process, sometimes a little superheat remains in the liquid when it first touches the mold wall, so solidification starts slightly below the top of the liquid level in the mold where $t = 0$. For other situations, the meniscus can solidify slightly so that the shell thickness at the top of the liquid level, $t = 0$, is already slightly positive. Both cases of the initial solidification front position s_0 are reasonable approximations usable in a simple model. In this work, we take the case $s_0 \geq 0$.

One useful consequence of assumption (A1) is that the Stefan condition (3) simplifies to

$$s_t = \frac{k}{\rho L_f} T_x\left(s^-, t\right) \tag{7}$$

which becomes linear when the property function $k/\rho L_f$ acts as a single constant parameter.

Another assumption will be made for the BC:

(A2) $q(t)$ is a bounded piecewise continuous function.

This assumption is physically justified, since in the actual caster, the feasible range of the cooling water flow rates is bounded due to the physical limitation of the spray cooling system, so the possible flux at the surface is bounded as well, and also piecewise continuous.

Assumptions (A1) and (A2) admit application of the maximum principle [12], that a parabolic PDE solution attains its maximum value on the boundaries of its spatiotemporal domain, to (1)–(6) using the methodology and the results of [13] and [14], where maximum principle is applied to the Stefan problem with, respectively, Neumann and Dirichlet BCs imposed at both liquid and solid boundaries.

By invoking the maximum principle, assumptions (A1) and (A2) jointly imply the second useful consequence. Namely, (2) and (A1) together yield

$$T < T_f, \quad x \in (0, s)\,;\; T = T_f, \quad x \in (s, L) \qquad \forall t. \tag{8}$$

Because it is open to this type of analysis on the subdomains, the Stefan problem will be used in this paper for mathematical analysis.

Based on [3, p. 96, Th. 3], the following unique solution $(T(x,t), s(t))$ of the system (1)–(6) under known boundary heat flux $q(t)$ can be found by *Picard iteration method* with any set of IC $(T(0), s(0))$ that satisfies the IC assumption (A1):

$$\begin{aligned} T\left(x,t\right) = &\\ - a\int_o^t q\left(\tau\right) G\left(x,0,t-\tau\right) d\tau + &\int_0^{s_0} T_0\left(\xi\right) G\left(x,\xi,t\right) d\xi \\ - a\int_0^t T_f \frac{\partial}{\partial \xi} G\left(x, s\left(\tau\right), t-\tau\right) d\tau + &\int_0^t \left(aT_x\left(s\left(\tau\right),\tau\right)\right. \\ \left. + T_f\,\dot{s}\left(\tau\right)\right) G\left(x,s\left(\tau\right),t-\tau\right) d\tau & \end{aligned} \tag{9}$$

where Green's function $G(x, \xi, t) = Q(x, \xi, t) + Q(-x, \xi, t)$ and Q is given by

$$Q\left(x,\xi,t\right) = \frac{1}{2\sqrt{\pi t}} e^{-\frac{(x-\xi)^2}{4t}} \tag{10}$$

and

$$s\left(t\right) = s_0 + \int_0^t \dot{s}\left(\tau\right) d\tau. \tag{11}$$

The recalibration parameters A and c introduced in (14) are included into the solution (9)–(11) through $q(\tau)$. Expressions (9)–(11) are often used to verify the results of various numerical techniques applied to solving system (1)–(6).

B. Enthalpy Formulation

Another PDE that can also be used to model the 1-D slice, in fact the actual equation used in [2], is a generalized form of conservation of energy. Let the function $H(T)$ calculate the enthalpy—the thermodynamic internal energy—of the material at a temperature T, and $k(T)$ be the temperature-dependent conductivity. Then, conservation of energy for heat diffusion can be written as

$$\left(H\left(T\left(x,t\right)\right)\right)_t = \left(k\left(T\left(x,t\right)\right) T_x\left(x,t\right)\right)_x, \quad x \in (0, L). \tag{12}$$

The BCs remain the same as in (5) and (6), and assumption (A1) can be similarly stated to ensure the problem is physically realistic.

It can be shown [4] that (12) and (1)–(3) are actually equivalent in a weak sense. Suppose k is constant, and H is defined as

$$H\left(T\left(x,t\right)\right) = \rho\left(c_p + L_f \cdot 1\left(T\left(x,t\right) - T_f\right)\right) \tag{13}$$

where ρ and c_p are the constant density and specific heat respectively, and $1(\cdot)$ is the unit step function. Then the weak forms of (12) and (13), and (1)–(3) under assumption (A1) are the same.

Of course, since (12) will then involve taking the derivative of a step function, it will not have a solution in the classical sense. This makes the equation more difficult to analyze mathematically. However, since it does not explicitly require a moving boundary at s, it may be numerically modeled on a fixed computational domain, with (13) slightly regularized.

C. Heat Flux

For the example used in this paper, the BC (6) is often modeled as [5]

$$T_x\left(0,t\right) = q\left(t\right) = Au(t)^c\left(T\left(0,t\right) - T_\infty\right). \tag{14}$$

Here, $u(t)$ is the local spray water flux (flow rate through unit surface area) hitting the strand surface at time t, which is the main method of temperature control in a continuous caster. The term T_∞ is the temperature of the spray water, which is easily measured. The constants A and c are the only fitting parameters in the paper. In general, they vary along the caster and depend on the design of the caster and the cooling sprays, as well as the steel grade being cast and the casting conditions. These parameters are unknown *a priori* and are determined by calibrating dynamic model simulations through experimental plant data and/or separate tests on a dedicated rig.

D. Specific Heat Realization of Enthalpy Formulation

While (1)–(6), (9)–(11), and (12) and (13) provide a fundamental basis for analyzing Stefan problems, industrial applications call for representing more complex behavior. An alternative to (12) and (13) that is conceptually the same is to use an effective specific heat. Mollifying (13) slightly gives a similar function $H(T)^*$ that is differentiable. The effective specific heat is defined as the derivative of enthalpy

$$c_p^*\left(T\left(x,t\right)\right) = \frac{dH^*\left(T\right)}{dT}. \tag{15}$$

Then, applying the chain rule, (12) becomes:

$$\rho c_p^*\left(T\left(x,t\right)\right) T_t\left(x,t\right) = \left(k\left(T\left(x,t\right)\right) T_x\left(x,t\right)\right)_x. \tag{16}$$

Although (16) has the form of a simple transient heat conduction problem, the effective specific heat c_p is nonlinear, being much larger during phase changes than at other temperatures. Using (15) and (16), an effective specific-heat-based model is employed in [2] for a single slice. This approach permits better matching of process data in modeling alloys [11], since their solidification occurs over a range of temperatures and positions, rather than at a single point. Based on the latter, [11] presents an industrial grade simulator CONSENSOR, currently used in production [10]. This simulator is employed in Section IV-B to demonstrate the applicability of the results derived on the basis of (1)–(6) and (14) to the actual industrial production.

III. Previous Results

A. Continuous-in-Time Output Feedback for the Stefan Problem

In [6] and [7], two possible output injection rules are proposed for the Stefan problem. Both assume the measured surface temperature, denoted as

$$y(t) = T(0,t) \tag{17}$$

can be measured throughout the caster. Both injection rules work by introducing $\hat{T}(x,t)$, $\hat{s}(t)$, the solution to a slightly modified version of (1)–(6).

In [6], instead of the heat flux BC (6), the Dirichlet BC

$$\hat{T}(0,t) = y(t) = T(0,t) \tag{18}$$

is used. This BC simply forces the estimator to exactly match the measured surface temperature. This rule was proven to be stable, but not necessarily convergent. Convergence was seen in simulation, as shown in Fig. 2(a).

In [7], instead of changing the BC, output is injected through the Stefan condition. Then, (3) changes to the estimate

$$\hat{s}_t = -\frac{k}{\rho L_f}\hat{T}_x\Big|_{x=s^-}^{x=s^+} + L\left(\hat{T}(0,t) - y(t)\right). \tag{19}$$

This is left as a conjecture and convergence or stability is still unproven. However, in simulation, it performs better than (18), showing what appears to be exponential convergence. An example simulation is shown in Fig. 2(b).

These approaches have the advantage of being based on a fundamental mathematical analysis of the problem, even with no explicit proof existing at present. However, they assume more sensing than is typically available in the actual physical system, and without drastic improvements in sensing technology they will not provide a widely implementable solution for the problem.

B. Open-Loop "Software Sensor" Estimator for Quasi-Linear Conservation of Energy

Within the steel industry, the pervasive problem of unreliable and sparse sensing has led to the widespread use of open-loop strand temperature control in the spray zone. In fact, control systems in the industry are still open-loop in nature, albeit quite sophisticated: temperature "feedback" is obtained from real-time computational models instead of physical sensors [8]–[10]. For example, [10] uses the real-time mold cooling measurements in a state-of-the-art software sensor that gives small temperature error at the mold exit. Thus, although these "software sensors" do take as inputs a wide range of measurements of casting conditions, the signal directly affected by the temperature of the strand itself in response to spray cooling is still unavailable to close the loop on the sprays through measurements taken in the spray zone. As a result, the estimate error, fairly small at the mold exit, grows as the distance from the latter increases.

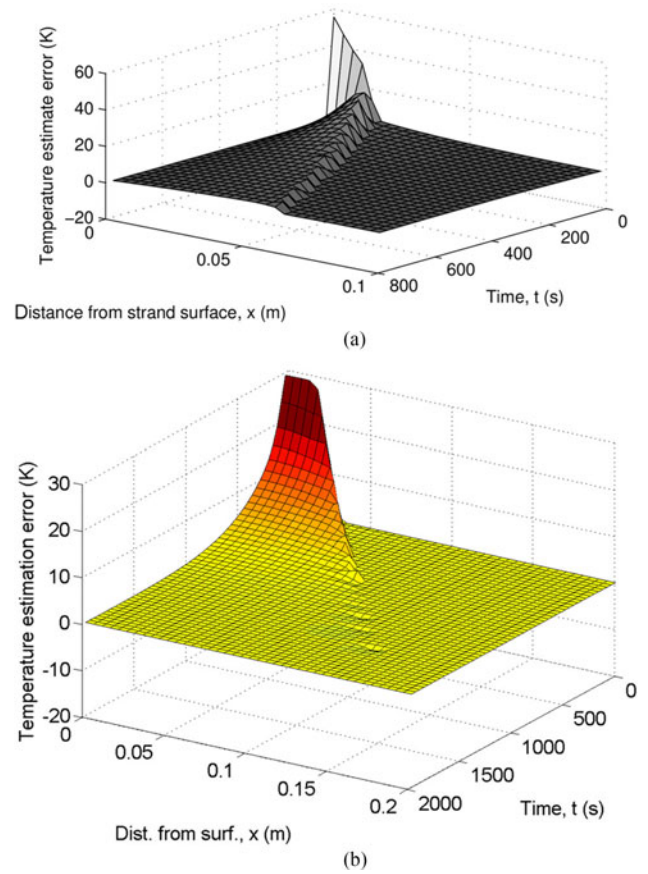

Fig. 2. Estimation error of Stefan problem estimators, using boundary sensing and two different estimation laws. (a) Estimation scheme (18) [6]. (b) Estimation scheme (19) [7].

IV. Discrete-in-Time Calibration

The purpose of this paper, then, is to start bridging the gap between the theoretical, but impractical approach of Section III-A, and the practical but unproven approach of Section III-B. The software sensors described in the previous section are open-loop in nature with respect to the spray zone. The only measurements available are not affected by the strand temperature—the output to be controlled through spray actuation. In fact, one of the problems the software sensor itself was specifically developed to solve is the lack of distributed temperature measurement in the strand. It is the nature of the problem that the only temperature measurements available, sparsely located pyrometers, are discrete-in-time and therefore do not lend themselves to standard estimation techniques.

However, one could attempt to "calibrate" the model—adjust its parameters to match sparse measurements—a common modeling problem. In general, this calibration problem assumes that the system dynamics are known except for a finite set of unknown parameters. In the present case of continuous casters, the least well-known parameters are those related to the boundary heat flux.

A. Calibration of the Stefan Problem

Although the Stefan problem is nonlinear, it is still parabolic in most of the strand. In addition, the unknown parameters in the boundary heat flux (14) all affect the heat flux monotonically: increasing the parameter increases the heat flux. Therefore, we can apply the well-known properties of parabolic PDEs, in particular the maximum principle, to the error equations.

Lemma 1: Let $T_1(x,t)$, $s_1(t)$ and $T_2(x,t)$, $s_2(t)$ be the solutions to (1)–(6), with the ICs $T_1(x,0) = T_2(x,0)$ and $s_1(0) \geq s_2(0)$ that satisfy the IC assumption (A1) and the BC assumption (A2), and have the same material properties a and $\frac{k}{\rho L_f}$. Then, if the boundary heat fluxes satisfy

$$q_1(t) > q_2(t) \quad \forall t$$

then the interface locations satisfy

$$s_2(t) \leq s_1(t) \quad \forall t \in (0,\ t_L)$$

where $s_2(t_L) = L$.

Proof: Suppose first that $s_1(0) > s_2(0)$. Then, $s_2(t) < s_1(t)$. If not, then there exists the first time $\bar{t}$ such that $s_2(\bar{t}) = s_1(\bar{t})$. Denote $\bar{t} = \inf\{t | s_2(t) = s_1(t)\}$.

Then, for $v(x,t) = T_2(x,t) - T_1(x,t)$, we have:

$$v_t = a v_{xx}, \quad x \in (0, s_2), \tag{20}$$

$$v_x(0,t) = q_2(t) - q_1(t) < 0. \tag{21}$$

By weak maximum principle [12, p. 389, Th. 8], $v(x,t)$ attains minimum value on parabolic boundary, i.e., minimum value attained on $t = 0$ or $x = s_2(t)$. Therefore

$$v \geq 0 \quad \forall x \in (0, s_2(\bar{t})) \quad \forall t \in (0, \bar{t}]. \tag{22}$$

If we have $v(x^*, t^*) = 0$ for $(x^*, t^*) \in (0, s_2(\bar{t})) \times (0, \bar{t}]$, invoking the strong maximum principle [12, p. 397, Th. 12] we would obtain $v \equiv 0$ in $(0, s_2(\bar{t})) \times (0, t^*)$. This is inconsistent with (21). Thus, we have

$$v(x,t) > 0 \quad \forall x \in (0, s_2(\bar{t})) \quad \forall t \in (0, \bar{t}]. \tag{23}$$

Then, v attains a minimum at $(s_2(\bar{t}), \bar{t}) = (s_1(\bar{t}), \bar{t})$, where

$$v(s_2(\bar{t}), \bar{t}) = 0. \tag{24}$$

Following [14], by parabolic version of Hopf's Lemma

$$\frac{\partial T_2}{\partial x}(s_2^-(\bar{t}), \bar{t}) < \frac{\partial T_1}{\partial x}(s_2^-(\bar{t}), \bar{t}) \Rightarrow v_x(s_2^-(\bar{t}), \bar{t}) < 0. \tag{25}$$

Now, due to (7) we have

$$v_x(s_2^-(\bar{t}), \bar{t}) = s_{2t}(\bar{t}) - s_{1t}(\bar{t}) < 0 \tag{26}$$

which yields

$$s_{2t}(\bar{t}) < s_{1t}(\bar{t}) \tag{27}$$

which is inconsistent with the definition of $\bar{t}$.

Now, suppose $s_1(0) = s_2(0)$. Let $T_\varepsilon(x,t)$, $s_\varepsilon(t)$ be the solution to (1)–(6), with the ICs q_1, $T_1(0)$, $s_1(0) + \varepsilon$. Then, from the first part of the proof, we have $s_2 < s_\varepsilon$, $s_1 < s_\varepsilon$.

And due to [13, Th. 2], we have

$$|s_1(t) - s_\varepsilon(t)| \leq C_1 \varepsilon \tag{28}$$

where C_1 is a constant which depends on temperature and material properties. Therefore

$$s_2(t) < s_\varepsilon(t) \leq s_1(t) + \varepsilon. \tag{29}$$

Now, upon letting $\varepsilon \to 0$, we recover $s_2(t) \leq s_1(t)$. ∎

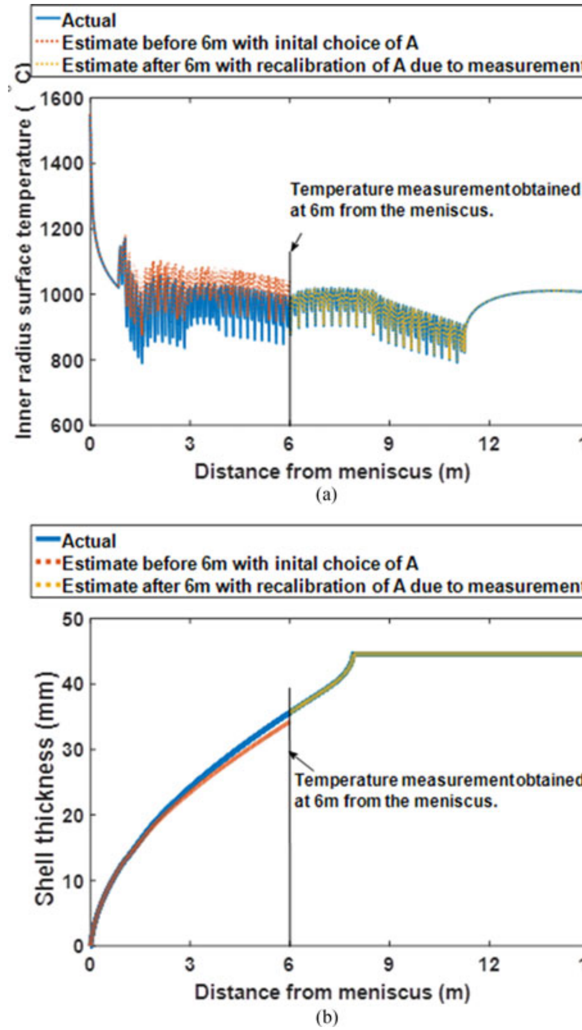

Fig. 3. Effective specific heat based simulation showing midsimulation calibration to match surface temperature measurement. (a) Surface temperature $T(0,t)$. (b) Shell thickness $s(t)$.

Lemma 2: Let $T_1(x,t)$, $s_1(t)$ and $T_2(x,t)$, $s_2(t)$ be the solutions to (1)–(6), with the same ICs $T_1(x,0) = T_2(x,0)$ and $s_1(0) = s_2(0)$ that satisfy the IC assumption (A1) and the BC assumption (A2), and have the same material properties a and b. Then, if the boundary heat fluxes satisfy

$$q_1(t) > q_2(t) \quad \forall t$$

then the temperatures satisfy

$$T_1(x,t) < T_2(x,t) \quad \forall t,\ \forall x \in [0,\ s_1).$$

Proof: According to Lemma 1, $s_2(t) \leq s_1(t) \quad \forall t$. Using weak and strong maximum principle as in Lemma 1 proof yields

$$T_1(x,t) \langle T_2(x,t)\ \forall t \rangle 0 \quad \forall x \in [0,\ s_2). \tag{30}$$

On $[s_2, s_1)$, trivially by (8) and the maximum principle for T_1, we must have

$$T_1 < T_f = T_2. \tag{31}$$

Combining (30) and (31) yields

$$T_1(x,t) < T_2(x,t) \quad \forall t, \forall x \in [0,\ s_1). \quad ∎$$

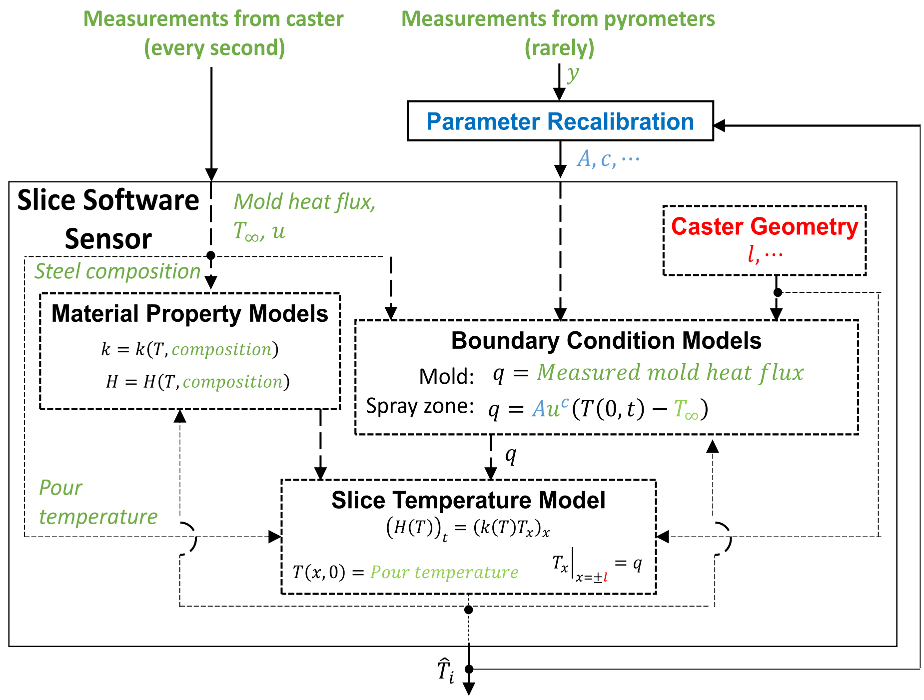

Fig. 4. Modified block diagram for slice software sensor using real-time parameter recalibration from pyrometer measurements.

An immediate consequence of this Lemma is that certain simple calibration problems must have a unique solution. For example, suppose the heat flux follows (14), assuming that only the parameter A is unknown. Under the physical assumptions on the other parameters and variables in (14), increasing A increases the heat flux. Then, due to Lemma 1, if the pour temperature and a measurement at any single other point in the caster are available, the actual value of A can be found exactly.

Theorem 1: Let $T(x,t)$ be the solution to the single-phase Stefan problem (1)–(6) under the assumption (A1) with the BC (14) satisfying assumption (A2), where either A or c is unknown. If the IC $T_0(x)$ and a measurement $T(x_{\text{measure}}, t_{\text{measure}})$, $x_{\text{measure}} \in [0,\ s_1)$, are known, the unknown parameter can be found to an arbitrary accuracy.

Proof: Applying Lemma 1 for a given IC T_0, since the heat flux (14)—and therefore the temperature gradient q—depends monotonically on A and c, the solution to the Stefan problem T will also depend monotonically on A and c. Therefore, there is a unique value of the unknown parameter that achieves a given measurement $T(x_{\text{measure}}, t_{\text{measure}})$. Furthermore, because of the monotonicity, it can be found, for example, by a simple binary search algorithm to any desired accuracy. ■

B. Discussion and Simulation

Clearly, this result provides a strong conclusion, but requires strict conditions. The heat flux does have a monotonic dependence on parameters A and c, but the parameters may vary throughout the caster. So, the assumption of a single missing parameter is not very likely.

Nevertheless, the technique presented is seen below to provide a practical framework for recalibration based on a single measurement. Indeed, Fig. 3 shows a simulation of an effective specific-heat-based PDE system CONSENSOR [11] based on (14)–(16), for which (1)–(6) and (14) can be viewed as a simplified approximation. The simulation is carried out under 3 m/min constant casting speed. The simulated steel composition and spray zone input values are those given in [11, Tables II and III]. The parameters k, ρ, L_f, and c_p vary with temperature T according to [2]. Until the pyrometer is reached, at 6 m from the meniscus (indicated in Fig. 1), the model assumes the value of A to be 1.57. In the actual system, this value is 2. At 6 m from the top of the caster, a measurement of the surface temperature is obtained, for example, from a pyrometer, a dragged thermocouple, or an infrared camera. A Newton search is used to adjust A to match the measurement. The derivative is calculated numerically using a simple finite-difference approximation. Within two iterations, the Newton search returns a value of 1.988 for A. After setting the predicted temperature profile at 6 m to the predicted value from CONSENSOR using this value of A, the simulation is restarted and continues for the rest of the caster. As seen in Fig. 3(b), the surface temperature and the shell thickness estimation errors are practically eliminated after recalibration at 6 m distance from the meniscus.

Fig. 4 shows the aggregated information flow of the software sensor discussed in Section II-B that is augmented by Section II-C to include parameter recalibration. The parameters A and c in (14) are updated as pyrometer measurements are taken for the slice. Compared with the setting in [11] that is open-loop with respect to the spray zone, the proposed estimator structure is no longer completely open-loop in this zone, and is able to sparsely spatiotemporally close the secondary cooling loop when a temperature measurement is received.

V. Conclusion and Future Work

In this paper, a methodology for recalibration of online estimator through discrete-in-time temperature measurements to reduce the error caused by a single unknown parameter in the model is proposed. It is conjectured that changing the assumption from $T_0(x) = T_f, \quad x \in (s_0, L)$ to $T_0(x) > T_f, \quad x \in (s_0, L)$, i.e., to the two phase Stefan problem, the result could apply to measurements in $[s_1,\ L]$. This will be examined in future work.

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